

$$26.) \frac{\sqrt{17}}{\sqrt{12}} \cdot \frac{\sqrt{17}}{\sqrt{12}} \cdot \frac{\sqrt{12}}{\sqrt{12}}$$

$$\frac{\sqrt{204}}{12}$$

$$\frac{\sqrt{4 \cdot 51}}{12} = \frac{2\sqrt{51}}{12} = \boxed{\frac{\sqrt{51}}{6}}$$

$$28.) \frac{2}{4+\sqrt{11}} \cdot \frac{4-\sqrt{11}}{4-\sqrt{11}}$$

$$\frac{8-2\sqrt{11}}{16-11}$$

$$\boxed{\frac{8-2\sqrt{11}}{5}}$$

$$27.) \frac{\sqrt{6}}{\sqrt{5}} \cdot \frac{\sqrt{6}}{\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}}$$

$$\boxed{\frac{\sqrt{30}}{5}}$$

$$29.) \frac{4}{8-\sqrt{3}} \cdot \frac{8+\sqrt{3}}{8+\sqrt{3}}$$

$$\frac{32+4\sqrt{3}}{64-3}$$

$$\boxed{\frac{32+4\sqrt{3}}{61}}$$

Solve the equation for x. Write your answer in simplest radical form.

$$30.) \frac{5x^2}{5} = \frac{80}{5}$$

$$x^2 = 16$$

$$\sqrt{x^2} = \pm \sqrt{16}$$

$$\boxed{x = \pm 4}$$

$$31.) x^2 = 84$$

$$\sqrt{x^2} = \pm \sqrt{84} < \frac{\sqrt{4}}{\sqrt{21}}$$

$$\boxed{x = \pm 2\sqrt{21}}$$

$$32.) 7x^2 - 10 = 25$$

$$+10 \quad +10$$

$$\frac{7x^2}{7} = \frac{35}{7}$$

$$x^2 = 5$$

$$\sqrt{x^2} = \pm \sqrt{5}$$

$$\boxed{x = \pm \sqrt{5}}$$

$$33.) \left(\frac{1}{3}(x-4)^2\right) = (11)^3$$

$$(x-4)^2 = 33$$

$$\sqrt{(x-4)^2} = \pm \sqrt{33}$$

$$x-4 = \pm \sqrt{33}$$

$$\boxed{x = 4 \pm \sqrt{33}}$$

$$34.) 2(x+2)^2 - 5 = 8$$

$$2(x+2)^2 = 13$$

$$(x+2)^2 = \frac{13}{2}$$

$$\sqrt{(x+2)^2} = \pm \sqrt{\frac{13}{2}}$$

$$x+2 = \pm \sqrt{\frac{13}{2}} \rightarrow \frac{\sqrt{13}}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{26}}{2}$$

$$\boxed{x = -2 \pm \frac{\sqrt{26}}{2}}$$

35.) The path of a basketball thrown at an angle of  $45^\circ$  can be modeled by  $y = -0.02x^2 + x + 6$ .

a.) What is the maximum height of the basketball?

$$\text{vertex: } (25, 18.5)$$

$$\boxed{18.5 \text{ ft}}$$

b.) What height is the basketball thrown from?

$$y\text{-int: } (0, 6)$$

$$\boxed{6 \text{ ft}}$$

$$x = \frac{-b}{2a} = \frac{-(1)}{2(-0.02)} = 25$$

$$y = -0.02(25)^2 + 25 + 6$$

$$y = 18.5$$

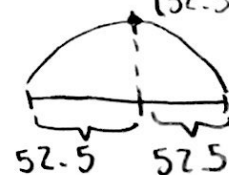
36.) The arch of the Gateshead Millennium Bridge forms a parabola with equation

$y = -0.016(x - 52.5)^2 + 45$  where  $x$  is the horizontal distance (in meters) from the arch's left end and  $y$  is the distance (in meters) from the base of the arch.

What is the width of the arch? vertex:  $(52.5, 45)$

$$52.5 + 52.5$$

$$\boxed{105 \text{ m}}$$

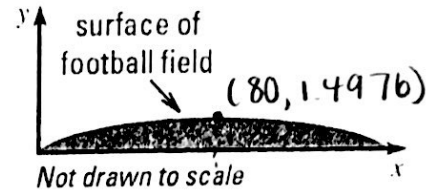


37.) Although a football field appears to be flat, its surface is actually shaped like a parabola so that rain runs off to both sides. The cross section of a field with synthetic turf can be modeled by

$$y = -0.000234x(x - 160)$$

where  $x$  and  $y$  are measured in feet.

vertex:  $\frac{p+q}{2}$   
 $\frac{0+160}{2} = 80$



a.) What is the field's width?

$$80 + 80 = \boxed{160 \text{ ft}}$$

b.) What is the maximum height of the field's surface?

$$y = -0.000234(80)(80 - 160)$$

$$y = 1.4976 \approx \boxed{1.5 \text{ ft}}$$

38.) An arch to the entrance of the library can be modeled by  $y = -0.13x^2 + 2.5x$  where  $x$  and  $y$  are measured in feet. To the nearest foot, what is the height of the highest point of the arch?

$$x = \frac{-b}{2a} = \frac{-2.5}{2(-0.13)} \approx 9.615$$

vertex:  $(9.615, 12.019)$

$$y = -0.13(9.615)^2 + 2.5(9.615)$$

$$y \approx 12.019 \approx \boxed{12 \text{ ft}}$$

39.) When an object is dropped, its height  $h$  (in feet) above the ground after  $t$  seconds can be modeled by the function.

$$h = -16t^2 + h_0$$

where  $h_0$  is the object's initial height (in feet).

A cliff diver dives off a cliff 40 feet above water.

a.) Write an equation giving the diver's height  $h$  (in feet) above the water after  $t$  seconds.

$$\boxed{h = -16t^2 + 40}$$

b.) How long is the diver in the air? (Round answers to the nearest tenth of a second)

$$0 = -16t^2 + 40 \rightarrow 2.5 = t^2$$

$$-40 = -16t^2 \rightarrow t = \pm 1.58 \approx \boxed{1.6 \text{ seconds}}$$

40.) The air resistance  $R$  (in pounds) on a racing cyclist is given by the equation  $R = 0.00829s^2$  where  $s$  is the bicycle's speed (in miles per hour).

What is the speed of a racing cyclist who experiences 5 pounds of air resistance?

$$5 = 0.00829s^2$$

$$s^2 \approx 603.136$$

$$\sqrt{s^2} \approx \pm \sqrt{603.136}$$

$$s \approx \pm 24.5588 \approx \boxed{24.6 \text{ mph}}$$

# Chapter 4 (Part 2) Review Worksheet

Name: KEY

Solve the equation.

1.)  $x^2 + 9 = 4$

$x^2 = -5$

$\sqrt{x^2} = \pm \sqrt{-5}$

$x = \pm i\sqrt{5}$

2.)  $x^2 = 2x^2 + 4$

$-x^2 = 4$

$x^2 = -4$

$\sqrt{x^2} = \pm \sqrt{-4}$

$x = \pm 2i$

3.)  $\frac{1}{3}x^2 + 10 = -23$

$\frac{1}{3}x^2 = -33$

$x^2 = -99$

$\sqrt{x^2} = \pm \sqrt{-99} \leftarrow \begin{matrix} i \\ \sqrt{9} \\ \sqrt{11} \end{matrix}$

$x = \pm 3i\sqrt{11}$

4.)  $-5x^2 - 3 = 97$

$-5x^2 = 100$

$x^2 = -20$

$\sqrt{x^2} = \pm \sqrt{-20} \leftarrow \begin{matrix} i \\ \sqrt{4} \\ \sqrt{5} \end{matrix}$

$x = \pm 2i\sqrt{5}$

5.)  $(x-10)^2 = -54$

$\sqrt{(x-10)^2} = \pm \sqrt{-54} \leftarrow \begin{matrix} i \\ \sqrt{9} \\ \sqrt{6} \end{matrix}$

$x-10 = \pm 3i\sqrt{6}$

$x = 10 \pm 3i\sqrt{6}$

6.)  $-(x+7)^2 + 8 = 44$

$-(x+7)^2 = 36$

$(x+7)^2 = -36$

$\sqrt{(x+7)^2} = \pm \sqrt{-36}$

$x+7 = \pm 6i$

$x = -7 \pm 6i$

Write the expression as a complex number in standard form.

7.)  $(8-6i) + (7+4i)$

$8+7-6i+4i$

$15-2i$

8.)  $(2-3i) - (6-5i)$

$2-6-3i+5i$

$-4+2i$

9.)  $(3+4i) - (2-5i)$

$3-2+4i+5i$

$1+9i$

10.)  $-9i(2-i) \quad * i^2 = -1$

$-18i + 9i^2$

$-18i - 9$

$-9-18i$

11.)  $(5+i)(4-2i)$

$20-10i+4i-2i^2$

$20-6i+2$

$22-6i$

12.)  $(2-7i)(-8-3i)$

$-16-6i+56i+21i^2$

$-16+50i-21$

$-37+50i$

$$13.) \frac{4i}{-3+6i} \cdot \frac{-3-6i}{-3-6i}$$

$$\frac{-12i - 24i^2}{9 - 36i^2}$$

$$\frac{-12i + 24}{9 + 36}$$

$$\frac{24 - 12i}{45}$$

$$\frac{24}{45} - \frac{12}{45}i = \boxed{\frac{8}{15} - \frac{4}{15}i}$$

$$14.) \frac{3+i}{2-3i} \cdot \frac{2+3i}{2+3i}$$

$$\frac{6+9i+2i+3i^2}{4-9i^2}$$

$$\frac{6+11i-3}{4+9}$$

$$\frac{3+11i}{13}$$

$$\boxed{\frac{3}{13} + \frac{11}{13}i}$$

$$15.) \frac{5+i}{7+4i} \cdot \frac{7-4i}{7-4i}$$

$$\frac{35-20i+7i-4i^2}{49-10i^2}$$

$$\frac{35-13i+4}{49+16}$$

$$\frac{39-13i}{65}$$

$$\frac{39}{65} - \frac{13}{65}i = \boxed{\frac{3}{5} - \frac{1}{5}i}$$

Use the properties of exponents to write the complex number in standard form.

$$16.) -5 + i^{74}$$

$i^1 = i$   
 $i^2 = -1$   
 $i^3 = -i$   
 $i^4 = 1$

$$\downarrow \begin{array}{l} 183 \\ 4 \\ 3 \end{array}$$

$$\boxed{-5 - i}$$

$$17.) 4 + i^{29}$$

$$\downarrow \begin{array}{l} 29 \\ 28 \\ 1 \end{array}$$

$$\boxed{4 + i}$$

$$18.) -11 - 2i^{66}$$

$$\downarrow \begin{array}{l} 66 \\ 60 \\ 6 \\ 2 \end{array}$$

$$-11 - 2(-1)$$

$$-11 + 2$$

$$\boxed{-9}$$

$$19.) 15 + 7i^{76}$$

$$\downarrow \begin{array}{l} 76 \\ 70 \\ 6 \\ 0 \end{array}$$

$$15 + 7(1)$$

$$15 + 7$$

$$\boxed{22}$$

Solve the equation by completing the square.  $(\frac{b}{2})^2$

$$20.) x^2 + 16x - 17 = 0 \quad (\frac{16}{2})^2 \rightarrow (8)^2 \rightarrow 64$$

$$x^2 + 16x + \boxed{64} = 17 + \boxed{64}$$

$$(x+8)^2 = 81$$

$$x+8 = \pm 9$$

$$x = -8 \pm 9$$

$$\boxed{x=1} \quad \boxed{x=-17}$$

$$21.) x^2 - 6x - 15 = 0 \quad (\frac{6}{2})^2 \rightarrow (3)^2 \rightarrow 9$$

$$x^2 - 6x + \boxed{9} = 15 + \boxed{9}$$

$$(x-3)^2 = 24$$

$$x-3 = \pm \sqrt{24} \leq \begin{array}{l} \sqrt{4} \\ \sqrt{6} \end{array}$$

$$\boxed{x = 3 \pm 2\sqrt{6}}$$

$$22.) \frac{2x^2 + 8x - 28}{2} = \frac{0}{2} \quad (\frac{4}{2})^2 \rightarrow (2)^2 \rightarrow 4$$

$$x^2 + 4x - 14 = 0$$

$$x^2 + 4x + \boxed{4} = 14 + \boxed{4}$$

$$(x+2)^2 = 18$$

$$x+2 = \pm \sqrt{18} \leq \begin{array}{l} \sqrt{9} \\ \sqrt{2} \end{array}$$

$$\boxed{x = -2 \pm 3\sqrt{2}}$$

$$23.) x^2 + 24x + 244 = 0 \quad (\frac{24}{2})^2 \rightarrow (12)^2 \rightarrow 144$$

$$x^2 + 24x + \boxed{144} = -244 + \boxed{144}$$

$$(x+12)^2 = -100$$

$$x+12 = \pm \sqrt{-100}$$

$$x+12 = \pm 10i$$

$$\boxed{x = -12 \pm 10i}$$

Write the quadratic function in vertex form. Then identify the vertex.

24.)  $y = x^2 + 14x + 39$   $(\frac{14}{2})^2 \rightarrow (7)^2 \rightarrow 49$   $\boxed{49} + y = x^2 + 14x + \boxed{49} + 39$   
 $49 + y = (x+7)^2 + 39$   
 $y = (x+7)^2 - 10$   
 Vertex:  $(-7, -10)$

25.)  $y = x^2 - 20x + 125$   $(\frac{20}{2})^2 \rightarrow (10)^2 \rightarrow 100$   $\boxed{100} + y = x^2 - 20x + \boxed{100} + 125$   
 $100 + y = (x-10)^2 + 125$   
 $y = (x-10)^2 + 25$   
 Vertex:  $(10, 25)$

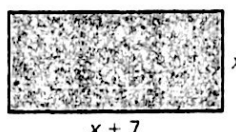
Find the value of x.

26.) Area of parallelogram = 48 units<sup>2</sup>  
 ( $A = b \cdot h$ )



$48 = x(x+6)$   $(\frac{6}{2})^2 \rightarrow (3)^2 \rightarrow 9$   
 $48 = x^2 + 6x$   
 $x^2 + 6x + \boxed{9} = 48 + \boxed{9}$   
 $(x+3)^2 = 57$   
 $x+3 = \pm\sqrt{57}$   $\boxed{x = -3 \pm \sqrt{57}}$

27.) Area of rectangle = 78 units<sup>2</sup>



$78 = x(x+7)$   $(\frac{7}{2})^2 \rightarrow (3.5)^2 \rightarrow 12.25$   
 $78 = x^2 + 7x$   
 $x^2 + 7x + \boxed{12.25} = 78 + \boxed{12.25}$   
 $(x + \frac{7}{2})^2 = 90.25$   
 $x + 3.5 = \pm 9.5$   
 $x = -3.5 \pm 9.5$

Use the quadratic equation to solve the equation.

28.)  $x^2 + 4x - 3 = 0$   
 $a \quad b \quad c$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$x = \frac{-4 \pm \sqrt{(4)^2 - 4(1)(-3)}}{2(1)}$   
 $x = \frac{-4 \pm \sqrt{28}}{2} < \frac{\sqrt{4}}{\sqrt{7}}$   
 $x = \frac{-4 \pm 2\sqrt{7}}{2}$   $\boxed{x = -2 \pm \sqrt{7}}$

29.)  $9x^2 = -6x - 1$

$9x^2 + 6x + 1 = 0$   
 $a \quad b \quad c$

$x = \frac{-6 \pm \sqrt{(6)^2 - 4(9)(1)}}{2(9)}$   
 $x = \frac{-6 \pm \sqrt{0}}{18}$   
 $x = \frac{-6}{18}$   $\boxed{x = -\frac{1}{3}}$

$\boxed{x = 0}$   ~~$x = 13$~~

30.)  $6x^2 - 8x = -3$

$6x^2 - 8x + 3 = 0$   
 $a \quad b \quad c$

$x = \frac{-(-8) \pm \sqrt{(-8)^2 - 4(6)(3)}}{2(6)}$

$x = \frac{8 \pm \sqrt{8}}{12} < \frac{\sqrt{4}}{\sqrt{2}}$

$x = \frac{8 \pm 2\sqrt{2}}{12}$   $\boxed{x = \frac{4 \pm \sqrt{2}}{6}}$

31.)  $3x^2 + 10x - 5 = 0$

$a \quad b \quad c$

$x = \frac{-10 \pm \sqrt{(10)^2 - 4(3)(-5)}}{2(3)}$

$x = \frac{-10 \pm \sqrt{160}}{6} < \frac{\sqrt{16}}{\sqrt{10}}$

$x = \frac{-10 \pm 4\sqrt{10}}{6}$

$x = \frac{-5 \pm 2\sqrt{10}}{3}$

32.) A person spikes a volleyball over a net when the ball is 9 feet above the ground. The volleyball has an initial vertical velocity of -40 feet per second. The volleyball is allowed to fall to the ground. How long is the ball in the air after it is spiked?

$$h = -16t^2 + v_0t + h_0 \quad t = \frac{-(-40) \pm \sqrt{(-40)^2 - 4(-16)(9)}}{2(-16)}$$

$$0 = -16t^2 - 40t + 9 \quad t = \frac{40 \pm \sqrt{2176}}{-32}$$

$$t \approx -2.21, 0.21 \approx \boxed{0.21 \text{ seconds}}$$

33.) A juggler tosses a ball into the air. The ball leaves the juggler's hand 4 feet above the ground and has an initial vertical velocity of 40 feet per second. The juggler catches the ball when it falls back to a height of 3 feet. How long is the ball in the air?

$$h = -16t^2 + v_0t + h_0 \quad t = \frac{-40 \pm \sqrt{(40)^2 - 4(-16)(1)}}{2(-16)}$$

$$3 = -16t^2 + 40t + 4 \quad t = \frac{-40 \pm \sqrt{1664}}{-32}$$

$$0 = -16t^2 + 40t + 1$$

$$t \approx -0.025, 2.525 \approx \boxed{2.5 \text{ seconds}}$$

34.)  $y \leq -2x^2 + 8x - 5$

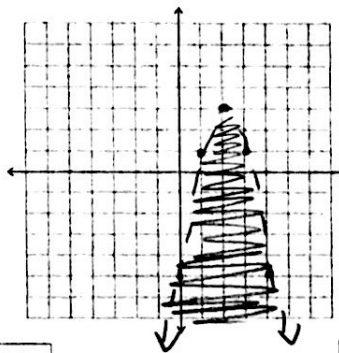
AOS:  $X = 2$

vertex:  $(2, 3)$

$$X = \frac{-8}{2(-2)} = \frac{-8}{-4} = 2$$

Test:  $\frac{x+4}{0/0} \times 0 \leq -5$

x	0	1	2	3	4
y	-5	1	3	1	-5



35.)  $y \geq -(x+5)(x+1)$

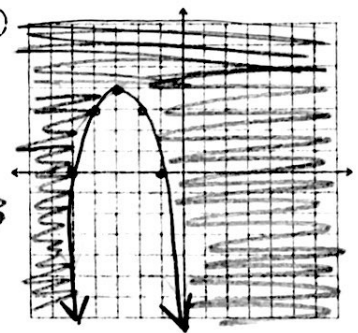
AOS:  $X = -3$

vertex:  $(-3, 4)$

$$X = \frac{-5+1}{2} = \frac{-4}{2} = -2$$

Test:  $\frac{x+4}{0/0} \times 0 \geq -5$

x	-5	-4	-3	-2	-1
y	0	3	4	3	0



36.)  $y \geq 2(x-4)^2 - 5$

$y \leq -x^2 + 4x + 2$

vertex:  $(4, -5)$

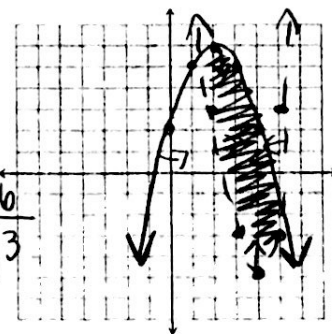
x	2	3	4	5	6
y	3	-3	-5	-3	3

Test:  $\frac{x+4}{0/0} \times 0 \leq 27$

$$X = \frac{-4}{2(-1)} = \frac{-4}{-2} = 2$$

vertex:  $(2, 6)$

x	0	1	2	3	4
y	2	5	6	5	2



37.)  $y \leq 2x^2 + 2$

$y \geq -x^2 - 3$

vertex:  $(0, 2)$

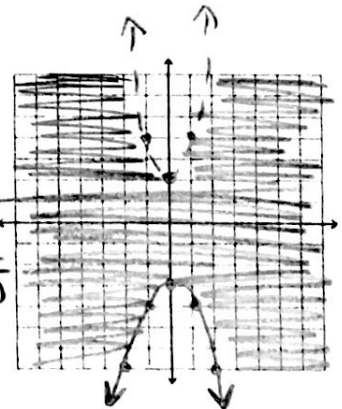
x	-2	-1	0	1	2
y	10	4	2	4	10

Test:  $\frac{x+4}{0/0} \times 0 \leq 2$

vertex:  $(0, -3)$

x	-2	-1	0	1	2
y	-7	-4	-3	-4	-7

Test:  $\frac{x+4}{0/0} \times 0 \geq -3$



Chapter 5 Review Worksheet

Name: KEY

Simplify the expression. Evaluate all powers with numerical bases. NO DECIMALS.

1.)  $(x^{-2}y^5)^2$   
 $x^{-4}y^{10}$   
 $\frac{y^{10}}{x^4}$

2.)  $(3x^4y^{-2})^{-3}$   
 $3^{-3}x^{-12}y^6$   
 $\frac{y^6}{3^3x^{12}}$   
 $\frac{y^6}{27x^{12}}$

3.)  $\frac{2x^{-6}y^5}{16x^3y^{-2}}$   
 $\frac{2x^{-9}y^7}{16}$   
 $\frac{y^7}{8x^9}$

4.)  $\frac{(3m^{-2}n^4)^{-3} \cdot m^{-6}}{9m^3n^{-3}} \cdot \frac{m^{-6}}{n^8}$   
 $\frac{3^{-3}m^6n^{-12}}{9m^3n^{-3}} \cdot \frac{m^{-6}}{n^8}$   
 $\frac{3^{-3}n^{-12}}{9m^3n^5}$   
 $\frac{n^{-17}}{9 \cdot 3^3 m^3}$   
 $\frac{1}{243m^3n^{17}}$

5.)  $\frac{5a^3}{(10b)^2} \cdot \frac{b^{-5}a^2}{a^7b^0}$   
 $\frac{5a^3}{100b^2} \cdot \frac{b^{-5}a^2}{a^7}$   
 $\frac{5a^5b^{-5}}{100a^7b^2}$   
 $\frac{a^{-2}b^{-7}}{20}$   
 $\frac{1}{20a^2b^7}$

6.)  $(2x^{-2}y^7)(12x^{-6}y^{-3})$   
 $24x^{-8}y^4$   
 $\frac{24y^4}{x^8}$

Decide whether the function is a polynomial function. If so, write it in standard form and state its degree, type, and leading coefficient. If it is not a polynomial, explain why.

7.)  $f(x) = x^4 - \frac{1}{4}x^2 + 3$  Yes  
 SF:  $f(x) = x^4 - \frac{1}{4}x^2 + 3$   
 D: 4  
 Type: Quartic  
 LC: 1

8.)  $h(x) = 5x^2 + 3x^{-1} - x$  No  
 can't have negative exponents

9.)  $g(x) = x + 2^x - 0.6x^5$  No  
 can't have variable exponents

10.)  $j(x) = 7x - \sqrt{3} + \pi x^2$  Yes  
 SF:  $j(x) = \pi x^2 + 7x - \sqrt{3}$   
 D: 2  
 Type: Quadratic  
 LC:  $\pi$

Evaluate the function for the given value of  $x$  using both direct and synthetic substitution.

11.)  $g(x) = 2x^4 - 5x^3 - 4x + 8$  when  $x = 3$

$$g(3) = 2(3)^4 - 5(3)^3 - 4(3) + 8$$

$$= 162 - 135 - 12 + 8$$

$$g(3) = 23$$

$$\begin{array}{r|rrrrr} 3 & 2 & -5 & 0 & -4 & 8 \\ & \downarrow & 6 & 3 & 9 & 15 \\ \hline & 2 & 1 & 3 & 5 & 23 \end{array}$$

$$g(3) = 23$$

12.)  $f(x) = x^5 - 2x^3 + 15$  when  $x = 4$

$$f(4) = (4)^5 - 2(4)^3 + 15$$

$$= 1024 - 128 + 15$$

$$f(4) = 911$$

$$\begin{array}{r|rrrrrr} 4 & 1 & 0 & -2 & 0 & 0 & 15 \\ & \downarrow & 4 & 16 & 56 & 224 & 896 \\ \hline & 1 & 4 & 14 & 56 & 224 & 911 \end{array}$$

$$f(4) = 911$$

Describe the end behavior of the graph of the polynomial function by completing the statements. (Hint: Sketch a general picture of the graph to help).

13.)  $f(x) = -8x^{10} + 21x^3$  D: even  
 $f(x) \rightarrow -\infty$  as  $x \rightarrow -\infty$  LC: -  
 $f(x) \rightarrow -\infty$  as  $x \rightarrow +\infty$  ↙ ↘

14.)  $f(x) = 12x^{15} - 2x^{14} + 8x^7 + 99$  D: odd  
 $f(x) \rightarrow -\infty$  as  $x \rightarrow -\infty$   
 $f(x) \rightarrow +\infty$  as  $x \rightarrow +\infty$  LC: + ↗ ↘

15.)  $f(x) = -x^5 + 1$  D: odd  
 $f(x) \rightarrow +\infty$  as  $x \rightarrow -\infty$  LC: -  
 $f(x) \rightarrow -\infty$  as  $x \rightarrow +\infty$  ↖ ↘

16.)  $f(x) = \frac{1}{2}x^6 + 8x^3 - 11x^2 + 19$  D: even  
 $f(x) \rightarrow +\infty$  as  $x \rightarrow -\infty$  LC: +  
 $f(x) \rightarrow +\infty$  as  $x \rightarrow +\infty$  ↖ ↗

Perform the indicated operation.

17.)  $(5x^3 - x + 3) + (x^3 - 9x^2 + 4x)$

$$6x^3 - 9x^2 + 3x + 3$$

18.)  $(x^3 + 4x^2 - 5x) - (4x^3 + x^2 - 7)$

$$-3x^3 + 3x^2 - 5x + 7$$



19.)  $(x-6)(5x^2+x-8)$

$x(5x^2+x-8) - 6(5x^2+x-8)$   
 $5x^3 + x^2 - 8x - 30x^2 - 6x + 48$

$5x^3 - 29x^2 - 14x + 48$

Factor the polynomial completely.

21.)  $64x^3 - 8$

$8(8x^3 - 1)$   
 $\uparrow \quad \uparrow$   
 $(2x)^3 \quad (1)^3$

\* difference of cubes

$8(2x-1)(4x^2+2x+1)$

23.)  $2x^3 - 7x^2 + 8x + 28$

$x^2(2x-7) - 4(2x-7)$

$(2x-7)(x^2-4)$

$(2x-7)(x+2)(x-2)$

20.)  $[(x-4)(x+7)](5x-1)$

$(x^2+7x-4x-28)(5x-1)$

$(5x-1)(x^2+3x-28)$

$5x(x^2+3x-28) - 1(x^2+3x-28)$

$5x^3 + 15x^2 - 140x - x^2 - 3x + 28$

$5x^3 + 14x^2 - 143x + 28$

22.)  $2x^5 - 12x^3 + 10x$

$2x(x^4 - 6x^2 + 5)$

$2x(x^2-5)(x^2-1)$

$2x(x^2-5)(x+1)(x-1)$

24.)  $27g^3 + 343$   
 $\uparrow \quad \uparrow$   
 $(3g)^3 \quad (7)^3$

\* sum of cubes

$(3g+7)(9g^2-21g+49)$

Find the real-number solutions of the equation (Start by factoring).

25.)  $16g^4 - 625 = 0$

$\uparrow \quad \uparrow$   
 $(4g^2)^2 \quad (25)^2$

$(4g^2+25)(4g^2-25) = 0$

$(4g^2+25)(2g+5)(2g-5) = 0$

$4g^2+25$   
 $g^2 = -25/4$   
 $\downarrow$   
 no real solution

$2g+5=0$   
 $g = -5/2$

$2g-5=0$   
 $g = 5/2$

26.)  $16x^3 - 44x^2 - 42x = 0$

$2x(8x^2 - 22x - 21) = 0$

$2x(8x^2 - 28x + 6x - 21) = 0$

$2x(4x(2x-7) + 3(2x-7)) = 0$

$2x(2x-7)(4x+3) = 0$

$2x=0$   
 $x=0$

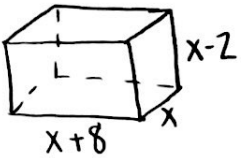
$2x-7=0$   
 $x=7/2$

$4x+3=0$   
 $x=-3/4$

$8 \cdot -21 = -168$   
 $-28 + 6 = -22$

27.) A shipping box is shaped like a rectangular prism. It has a total volume of 96 cubic inches. The height is two inches less than the width and the length is eight inches longer than the width.

a.) Write a polynomial equation in standard form that represents the volume of the box.



$$\begin{aligned} V &= x(x+8)(x-2) \\ &= x(x^2 - 2x + 8x - 16) \\ &= x(x^2 + 6x - 16) \end{aligned}$$

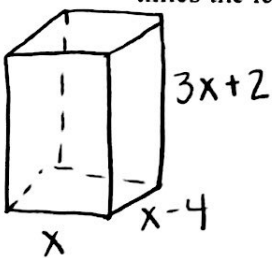
$$\boxed{V = x^3 + 6x^2 - 16x}$$

b.) Solve the polynomial equation from part a. What are the dimensions of the box?

$$\begin{aligned} 96 &= x^3 + 6x^2 - 16x \\ 0 &= x^3 + 6x^2 - 16x - 96 \\ 0 &= x^2(x+6) - 16(x+6) \\ 0 &= (x+6)(x^2 - 16) \\ 0 &= (x+6)(x+4)(x-4) \\ x &\neq -6 \quad x \neq -4 \quad \boxed{x=4} \end{aligned}$$

$$\boxed{\begin{array}{l} h: 2 \text{ in} \\ w: 4 \text{ in} \\ l: 12 \text{ in} \end{array}}$$

28.) You have 240 cubic inches of clay with which to make a sculpture shaped like a rectangular prism. You want the width to be 4 inches less than the length and the height to be 2 inches more than 3 times the length. What should the dimensions of the box be?



$$\begin{aligned} x(x-4)(3x+2) &= 240 \\ x(3x^2 + 2x - 12x - 8) &= 240 \\ x(3x^2 - 10x - 8) &= 240 \\ 3x^3 - 10x^2 - 8x &= 240 \\ 3x^3 - 10x^2 - 8x - 240 &= 0 \\ x^2(3x - 10) + 8(x - 30) &= 0 \\ &?? \end{aligned}$$

$\boxed{\text{We need an additional method to factor this!}}$

## Chapter 5 (Part 2) Review Worksheet

Name: KEY

Divide using polynomial long division.

1.)  $(x^2 + 5x - 14) \div (x - 2)$

$$\begin{array}{r} x+7 \\ x-2 \overline{) x^2 + 5x - 14} \\ \underline{-x^2 + 2x} \phantom{-14} \\ 7x - 14 \\ \underline{-7x + 14} \\ 0 \end{array}$$

$$\boxed{x+7}$$

2.)  $(6x^2 - 5x + 9) \div (2x - 1)$

$$\begin{array}{r} 3x-1 \\ 2x-1 \overline{) 6x^2 - 5x + 9} \\ \underline{-6x^2 + 3x} \phantom{+9} \\ -2x + 9 \\ \underline{-2x + 1} \\ 8 \end{array}$$

$$\boxed{3x-1 + \frac{8}{2x-1}}$$

3.)  $(5x^4 + 2x^3 - 9x + 12) \div (x^2 - 3x + 4)$

$$\begin{array}{r} 5x^2 + 17x + 31 \\ x^2 - 3x + 4 \overline{) 5x^4 + 2x^3 + 0x^2 - 9x + 12} \\ \underline{-5x^4 + 15x^3 - 20x^2} \phantom{+12} \\ 17x^3 - 20x^2 - 9x \phantom{+12} \\ \underline{-17x^3 + 51x^2 - 68x} \phantom{+12} \\ 31x^2 - 77x + 12 \\ \underline{-31x^2 + 93x - 124} \\ 10x - 112 \end{array}$$

$$\boxed{5x^2 + 17x + 31 + \frac{10x - 112}{x^2 - 3x + 4}}$$

4.)  $(4x^4 + 5x - 4) \div (x^2 - 3x - 2)$

$$\begin{array}{r} 4x^2 + 12x + 44 \\ x^2 - 3x - 2 \overline{) 4x^4 + 0x^3 + 0x^2 + 5x - 4} \\ \underline{-4x^4 + 12x^3 + 8x^2} \phantom{-4} \\ 12x^3 + 8x^2 + 5x \\ \underline{-12x^3 + 36x^2 + 24x} \phantom{-4} \\ -44x^2 + 29x - 4 \\ \underline{-44x^2 + 132x + 88} \\ 101x + 84 \end{array}$$

$$\boxed{4x^2 + 12x + 44 + \frac{101x + 84}{x^2 - 3x - 2}}$$

Divide using synthetic division.

5.)  $(x^4 - 7x^2 + 9x - 10) \div (x - 2)$

$$\begin{array}{r|rrrrr} 2 & 1 & 0 & -7 & 9 & -10 \\ & \downarrow & 2 & 4 & -6 & 6 \\ \hline & 1 & 2 & -3 & 3 & -4 \text{ R} \end{array}$$

$$\boxed{x^3 + 2x^2 - 3x + 3 + \frac{-4}{x-2}}$$

6.)  $(2x^2 - 11x^3 + 15x^2 + 6x - 18) \div (x - 3)$

$$\begin{array}{r|rrrr} 3 & -11 & 17 & 6 & -18 \\ & \downarrow & -33 & -48 & -126 \\ \hline & -11 & -10 & -42 & -144 \text{ R} \end{array}$$

$$\boxed{-11x^2 - 10x - 42 + \frac{-144}{x-3}}$$

Given polynomial  $f(x)$  and a factor of  $f(x)$ , factor  $f(x)$  completely.

7.)  $f(x) = x^3 - 3x^2 - 16x - 12; (x - 6)$

$$\begin{array}{r|rrrr} 6 & 1 & -3 & -16 & -12 \\ & \downarrow & 6 & 18 & 12 \\ \hline & 1 & 3 & 2 & 0 \end{array}$$

$$(x-6)(x^2+3x+2)$$

$$\boxed{(x-6)(x+1)(x+2)}$$

8.)  $f(x) = 3x^3 - 16x^2 - 103x + 36; (x + 4)$

$$\begin{array}{r|rrrr} -4 & 3 & -16 & -103 & 36 \\ & \downarrow & -12 & 112 & -36 \\ \hline & 3 & -28 & 9 & 0 \end{array}$$

$$(x+4)(3x^2-28x+9)$$

$$(x+4)(3x^2-27x-X+9)$$

$$(x+4)(3x(x-9)-1(x-9))$$

$$\boxed{(x+4)(x-9)(3x-1)}$$

$$3 \cdot 9 = 27$$

$$-27 \cdot -1 = -28$$

Given polynomial function  $f$  and a zero of  $f$ , find the other zeros of the function.

9.)  $f(x) = 2x^3 + 3x^2 - 39x - 20; \text{ zero: } 4$

$$\begin{array}{r|rrrr} 4 & 2 & 3 & -39 & -20 \\ & \downarrow & 8 & 44 & 20 \\ \hline & 2 & 11 & 5 & 0 \end{array}$$

$$(x-4)(2x^2+11x+5) = 0$$

$$(x-4)(2x^2+10x+X+5) = 0$$

$$(x-4)(2x(x+5)+1(x+5)) = 0$$

$$(x-4)(x+5)(2x+1) = 0$$

$$\boxed{x=4} \quad \boxed{x=-5} \quad \boxed{x=-\frac{1}{2}}$$

$$2 \cdot 5 = 10$$

$$10 \cdot 1 = 11$$

10.)  $f(x) = x^3 - 9x^2 - 5x + 45; \text{ zero: } 9$

$$\begin{array}{r|rrrr} 9 & 1 & -9 & -5 & 45 \\ & \downarrow & 9 & 0 & -45 \\ \hline & 1 & 0 & -5 & 0 \end{array}$$

$$(x-9)(x^2-5) = 0$$

$$\boxed{x=9}$$

$$x^2=5$$

$$\boxed{x=\pm\sqrt{5}}$$

Find all real zeros of the function.

11.)  $h(x) = x^3 + 4x^2 + x - 6$

$$\pm 1, \pm 2, \pm 3, \pm 6$$

$$\begin{array}{r|rrrr} 1 & 1 & 4 & 1 & -6 \\ & \downarrow & 1 & 5 & 6 \\ \hline & 1 & 5 & 6 & 0 \end{array}$$

$$(x-1)(x^2+5x+6) = 0$$

$$(x-1)(x+3)(x+2) = 0$$

$$\boxed{x=1} \quad \boxed{x=-3} \quad \boxed{x=-2}$$

12.)  $g(x) = x^3 - 5x^2 - 18x + 72$

$$\pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 8, \pm 9, \pm 12, \pm 18, \pm 24, \pm 36, \pm 72$$

$$\begin{array}{r|rrrr} 3 & 1 & -5 & -18 & 72 \\ & \downarrow & 3 & -6 & -72 \\ \hline & 1 & -2 & -24 & 0 \end{array}$$

$$(x-3)(x^2-2x-24) = 0$$

$$(x-3)(x-6)(x+4) = 0$$

$$\boxed{x=3} \quad \boxed{x=6} \quad \boxed{x=-4}$$

Find all real zeros of the function.

13.)  $f(x) = 2x^3 + 4x^2 - 2x - 4$

$\pm 1, \pm 2, \pm 4, \pm \frac{1}{2}$

$$\begin{array}{r|rrrr} 1 & 2 & 4 & -2 & -4 \\ & \downarrow & & & \\ & 2 & 6 & 4 & 0 \end{array}$$

$(x-1)(2x^2+6x+4)=0$

$(x-1)(2x^2+2x+4x+4)=0$

$(x-1)(2x(x+1)+4(x+1))=0$

$(x-1)(x+1)(2x+4)=0$

$x=1$     $x=-1$     $x=-2$

Find all zeros of the polynomial function.

15.)  $f(x) = x^4 + 4x^3 + 7x^2 + 16x + 12$

$\pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 12$

(-1)  $\begin{array}{r|rrrrr} 1 & 1 & 4 & 7 & 16 & 12 \\ & \downarrow & & & & \\ & 1 & 3 & 4 & 12 & 0 \end{array}$

$(x+1)(x+3)(x^2+4)=0$

$x=-1$     $x=-3$

$x^2 = -4$

$x = \pm 2i$

(-3)  $\begin{array}{r|rrrr} 1 & 1 & 3 & 4 & 12 \\ & \downarrow & & & \\ & 1 & 0 & 4 & 0 \end{array}$

$x^2 + 4$

Write a polynomial function  $f$  of least degree that has rational coefficients, a leading coefficient of 1, and the given zeros.

17.)  $-3, -1, -2i$

$(x+3)(x+1)(x-2i)(x+2i)$

$(x^3+x+3x+3)(x^2+2(x-2i)x-4i^2)$

$(x^2+4x+3)(x^2+4)$

$x^2(x^2+4x+3) + 4(x^2+4x+3)$

$x^4 + 4x^3 + 3x^2 + 4x^2 + 16x + 12$

$x^4 + 4x^3 + 7x^2 + 16x + 12$

14.)  $g(x) = 2x^3 - 5x^2 - 14x + 8$

$\pm 1, \pm 2, \pm 4, \pm 8, \pm \frac{1}{2}$

-2  $\begin{array}{r|rrrr} 2 & 2 & -5 & -14 & 8 \\ & \downarrow & & & \\ & 2 & -9 & 4 & 0 \end{array}$

$(x+2)(2x^2-9x+4)=0$

$(x+2)(2x^2-8x+x+4)=0$

$(x+2)(2x(x-4)-1(x-4))=0$

$(x+2)(x-4)(2x-1)=0$

$x=-2$     $x=4$     $x=\frac{1}{2}$

16.)  $g(x) = x^4 + 5x^3 - 7x^2 - 29x + 30$

$\pm 1, \pm 2, \pm 3, \pm 5, \pm 6, \pm 10, \pm 15, \pm 30$

(1)  $\begin{array}{r|rrrrr} 1 & 1 & 5 & -7 & -29 & 30 \\ & \downarrow & & & & \\ & 1 & 6 & -1 & -30 & 0 \end{array}$

$x^3+6x^2-x-30$

$(x-1)(x-2)$

(2)  $\begin{array}{r|rrrr} 1 & 1 & 6 & -1 & -30 \\ & \downarrow & & & \\ & 1 & 8 & 15 & 0 \end{array}$

$(x^2+8x+15)=0$

$(x-1)(x-2)$

$(x+5)(x+3)=0$

$x^2+8x+15$

$x=1$     $x=2$   
 $x=-5$     $x=-3$

18.)  $3, 2+\sqrt{3}$

$(x-3)(x-(2+\sqrt{3}))(x-(2-\sqrt{3}))$

$(x-3)((x-2)-\sqrt{3})((x-2)+\sqrt{3})$

$(x-3)(x-2)^2 + \sqrt{3}(x-2) - \sqrt{3}(x-2) - 3$

$(x-3)(x^2-2x-2x+4-3)$

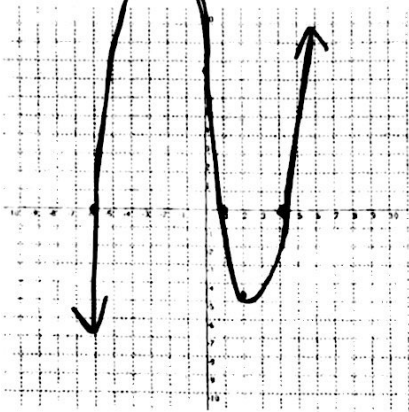
$(x-3)(x^2-4x+1)$

$x(x^2-4x+1) - 3(x^2-4x+1)$

$x^3-4x^2+x-3x^2+12x-3$

$x^3-7x^2+13x-3$

19.)  $h(x) = 0.3(x+6)(x-1)(x-4)$

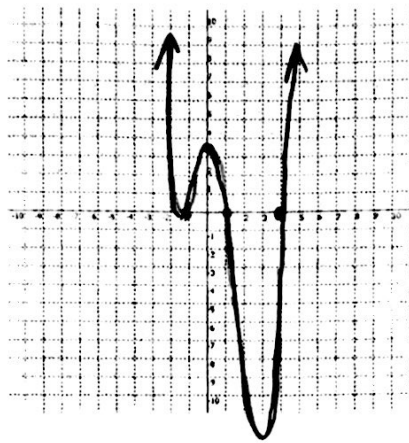


x-intercept(s):  $(-6, 0)$   $(1, 0)$   $(4, 0)$

y-intercept:  $(0, 7.2)$

x	-7	-5	-4	-1	2	3	5
y	-26.4	16.2	24	15	-4.8	-5.4	13.2

20.)  $f(x) = \frac{5}{6}(x+1)^2(x-1)(x-4)$

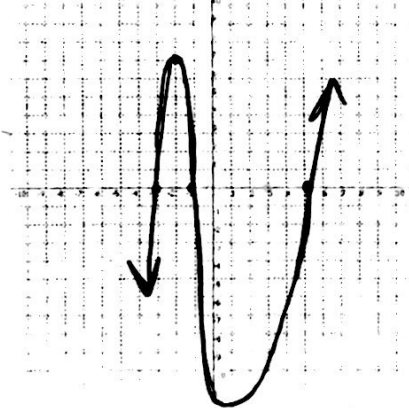


x-intercept(s):  $(-1, 0)$   $(1, 0)$   $(4, 0)$

y-intercept:  $(0, 3.3)$

x	-2	2	3	5			
y	15	-15	-26.7	120			

21.)  $h(x) = x^3 - x^2 - 17x - 15$   
 $(x+1)(x-5)(x-3)$



x-intercept(s):  $(-1, 0)$   $(5, 0)$   $(-3, 0)$

y-intercept:  $(0, -15)$

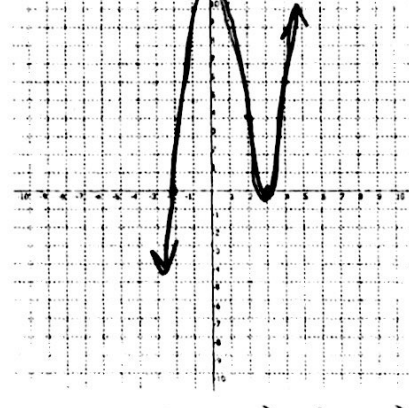
x	-4	-2	1	2	3	4	6
y	-27	7	-32	-45	-48	-35	63

$\pm 1, \pm 3, \pm 5, \pm 15$   $(x+1)(x^2-2x-15)$

$-1 \mid 1 \ -1 \ -17 \ -15$   $(x+1)(x-5)(x+3)$

$$\begin{array}{r} -1 \mid 1 \ -1 \ -17 \ -15 \\ \downarrow -1 \ 2 \ 15 \\ \hline 1 \ -2 \ -15 \ 0 \end{array}$$

22.)  $f(x) = x^3 - 4x^2 - 3x + 18$   
 $(x+2)(x-3)^2$



x-intercept(s):  $(-2, 0)$   $(3, 0)$

y-intercept:  $(0, 18)$

x	-3	-1	1	2	4	5	
y	-36	16	12	4	6	28	

$\pm 1, \pm 2, \pm 3, \pm 6, \pm 9, \pm 18$

$-2 \mid 1 \ -4 \ -3 \ 18$

$$\begin{array}{r} -2 \mid 1 \ -4 \ -3 \ 18 \\ \downarrow -2 \ 12 \ -18 \\ \hline 1 \ -6 \ 9 \ 0 \end{array}$$

$(x+2)(x^2-6x+9)$   
 $(x+2)(x-3)^2$