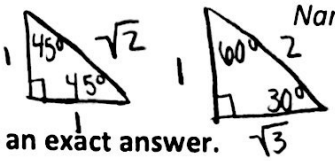
Chapter 13 Review Worksheet

Name: KEY

NO CALCULATOR, NO UNIT CIRCLE



Evaluate the trigonometric function. Give an exact answer.

1.) $\csc \frac{\pi}{6}$ $30°$ (H/O)

$\csc \frac{\pi}{6} = \frac{2}{1} = \boxed{2}$

2.) $\sec \frac{\pi}{3}$ $60°$ (H/A)

$\sec \frac{\pi}{3} = \frac{2}{1} = \boxed{2}$

3.) $\cos \frac{\pi}{4}$ $45°$ (A/H)

$\cos \frac{\pi}{4} = \frac{1}{\sqrt{2}} = \boxed{\frac{\sqrt{2}}{2}}$

4.) $\sin \frac{\pi}{3}$ $60°$ (O/H)

$\sin \frac{\pi}{3} = \boxed{\frac{\sqrt{3}}{2}}$

5.) $\cot \frac{\pi}{4}$ $45°$ (A/O)

$\cot \frac{\pi}{4} = \frac{1}{1} = \boxed{1}$

6.) $\tan \frac{\pi}{6}$ $30°$ (O/A)

$\tan \frac{\pi}{6} = \frac{1}{\sqrt{3}} = \boxed{\frac{\sqrt{3}}{3}}$

Evaluate the six trigonometric functions of θ.

7.) $\theta = 360°$

$\sin\theta = 0$

$\cos\theta = 1$

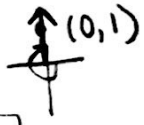
$\tan\theta = \frac{0}{1} = 0$

$\csc\theta = \frac{1}{0} = \text{UND}$

$\sec\theta = \frac{1}{1} = 1$

$\cot\theta = \frac{1}{0} = \text{UND}$

8.) $\theta = -\frac{3\pi}{2}$



$\sin\theta = 1$

$\cos\theta = 0$

$\tan\theta = \frac{1}{0} = \text{UND}$

$\csc\theta = \frac{1}{1} = 1$

$\sec\theta = \frac{1}{0} = \text{UND}$

$\cot\theta = \frac{0}{1} = 0$

NO CALCULATOR, MAY USE UNIT CIRCLE

Evaluate the function without using a calculator (i.e. ALL ANSWERS SHOULD BE EXACT, NO DECIMALS).

9.) $\tan 330°$ (y/x)

$\tan 330° = \frac{-\frac{1}{2}}{\frac{\sqrt{3}}{2}}$

$= -\frac{1}{2} \cdot \frac{2}{\sqrt{3}} = -\frac{1}{\sqrt{3}} = \boxed{-\frac{\sqrt{3}}{3}}$

10.) $\csc(-405°) \to 315°$ (1/y)

$\csc(-405°) = \frac{1}{-\frac{\sqrt{2}}{2}}$

$= 1 \cdot -\frac{2}{\sqrt{2}} = -\frac{2\sqrt{2}}{2} = \boxed{-\sqrt{2}}$

11.) $\tan 150°$ (y/x)

$\tan 150° = \frac{\frac{1}{2}}{-\frac{\sqrt{3}}{2}}$

$= \frac{1}{2} \cdot \frac{2}{-\sqrt{3}} = -\frac{1}{\sqrt{3}} = \boxed{-\frac{\sqrt{3}}{3}}$

12.) $\sec(-480°) \to 240°$ (1/x)

$\sec(-480°) = \frac{1}{-\frac{1}{2}}$

$= 1 \cdot -\frac{2}{1} = \boxed{-2}$

13.) $\sin \frac{13\pi}{6} \to 30°$ (y)

$\sin \frac{13\pi}{6} = \boxed{\frac{1}{2}}$

14.) $\sec \frac{11\pi}{3} \to 300°$ (1/x)

$\sec \frac{11\pi}{3} = \frac{1}{\frac{1}{2}}$

$= 1 \cdot \frac{2}{1} = \boxed{2}$

Domain → sin → $[-90°, 90°]$ cos → $[0°, 180°]$ tan → $(-90°, -90°)$

15.) $\cos^{-1} 1$ — $0°$, 0 radians

16.) $\tan^{-1} \sqrt{3}$ — $60°, \frac{\pi}{3}$

17.) $\sin^{-1}\left(-\frac{\sqrt{2}}{2}\right)$ — $-45°, -\frac{\pi}{4}$

18.) $\cos^{-1}\left(-\frac{\sqrt{3}}{2}\right)$ — $150°, \frac{5\pi}{6}$

19.) $\sin^{-1} 0$ — $0°$, 0 radians

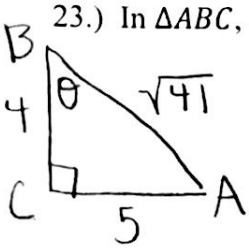
20.) $\cos^{-1} 3$ — undefined

21.) $\tan^{-1} 1$ — $45°, \frac{\pi}{4}$

22.) $\sin^{-1}\left(-\frac{1}{2}\right)$ — $-30°, -\frac{\pi}{6}$

MAY USE CALCULATOR, MAY USE UNIT CIRCLE AND FORMULAS

23.) In ΔABC, $a = 4, b = 5$, and $C = 90°$. Evaluate the six trigonometric functions of angle B.



$4^2 + 5^2 = c^2$

$c = \sqrt{41}$

$\frac{O}{H}$ $\sin\theta = \frac{5}{\sqrt{41}} = \frac{5\sqrt{41}}{41}$

$\frac{A}{H}$ $\cos\theta = \frac{4}{\sqrt{41}} = \frac{4\sqrt{41}}{41}$

$\frac{O}{A}$ $\tan\theta = \frac{5}{4}$

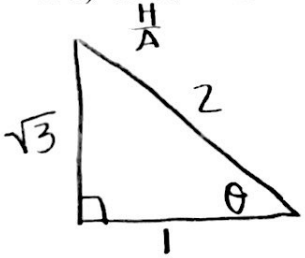
$\csc\theta = \frac{\sqrt{41}}{5}$

$\sec\theta = \frac{\sqrt{41}}{4}$

$\cot\theta = \frac{4}{5}$

Let θ be an acute angle of a right triangle. Find the values of the other five trigonometric fucntions of θ.

24.) $\sec\theta = 2$



$1^2 + x^2 = 2^2$

$x = \sqrt{3}$

$\frac{O}{H}$ $\sin\theta = \frac{\sqrt{3}}{2}$

$\frac{A}{H}$ $\cos\theta = \frac{1}{2}$

$\frac{O}{A}$ $\tan\theta = \frac{\sqrt{3}}{1} = \sqrt{3}$

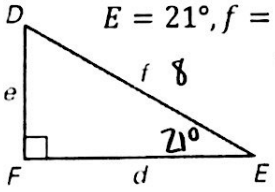
$\csc\theta = \frac{2}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$

$\sec\theta = \frac{2}{1} = 2$

$\cot\theta = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$

Solve ΔABC using the diagram and the given measurements. Round answers to the nearest tenth, when necessary.

25.) $E = 21°, f = 8$

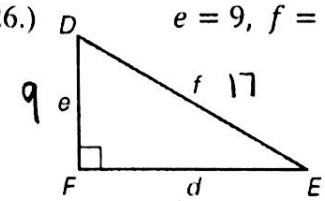


$\angle D = 69°$

$\sin 21° = \frac{e}{8}$ $\quad \cos 21° = \frac{d}{8}$

$e \approx 2.9$ $\quad d \approx 7.5$

26.) $e = 9, f = 17$



$9^2 + d^2 = 17^2$

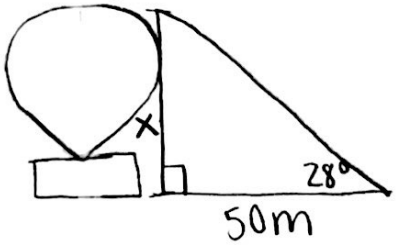
$d = \sqrt{208} \approx 14.4$

$\cos D = \frac{9}{17}$

$\angle D = \cos^{-1}\left(\frac{9}{17}\right)$

$\angle D \approx 58.0°$ $\quad \angle E \approx 32.0°$

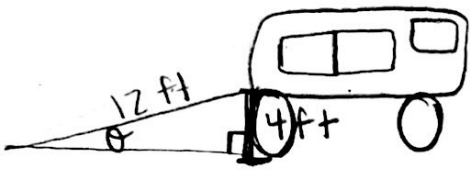
27.) You are standing 50 meters from a hot air balloon that is preparing to take off. The angle of elevation to the top of the balloon is 28°. Find the height of the balloon.



$\tan 28° = \frac{x}{50}$

$x \approx 26.6\text{m}$

28.) You use a 12 foot ramp to load items into a van. If the floor of the van is 4 feet off the ground, what is the angle of elevation of the ramp?



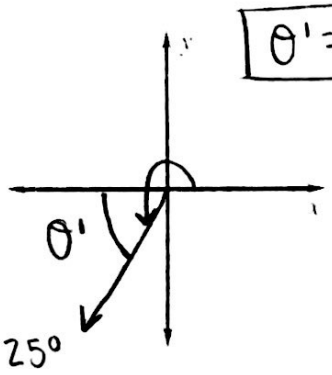
$\sin\theta = \frac{4}{12}$

$\theta = \sin^{-1}\left(\frac{4}{12}\right)$

$\theta \approx 19.5°$

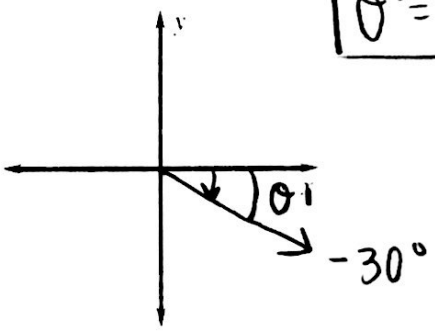
Sketch the angle. Then find its reference angle. Answer in the unit of the given angle.

29.) 250° θ'



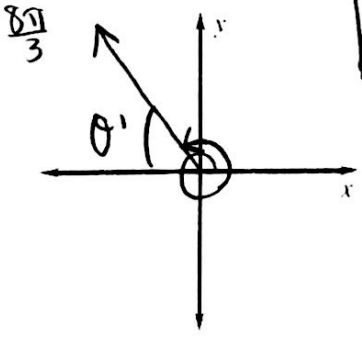
$\theta' = 70°$

30.) −30°



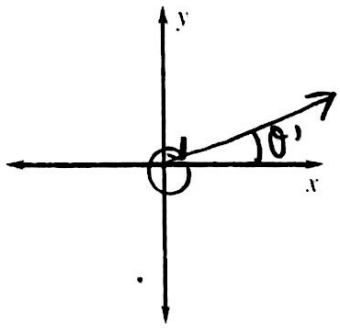
$\theta' = 30°$

31.) $\frac{8\pi}{3}$



$\theta' = \frac{\pi}{3}$

32.) $-\frac{11\pi}{6}$



$\theta' = \frac{\pi}{6}$

$\pm 360°$ OR $\pm 2\pi$

Find one positive angle and one negative angle that are coterminal with the given angle.

33.) $155° \pm 360°$ (+)

+: 515°, ...

−: −205°, ...

more possibilities!

34.) $-325° \pm 360°$ (−)

+: 35°, ...

−: −685°, ...

35.) $\frac{11\pi}{5} \pm \frac{10\pi}{5}$

+: $\frac{\pi}{5}, \frac{21\pi}{5}, \ldots$

−: $\frac{-9\pi}{5}, \ldots$

36.) $\frac{15\pi}{7} \pm \frac{14\pi}{7}$

+: $\frac{\pi}{7}, \frac{29\pi}{7}, \ldots$

−: $\frac{-13\pi}{7}, \ldots$

Convert the degree measure to radians or the radian measure to degrees.

37.) $145°$

$145°\left(\frac{\pi}{180°}\right)$

$\frac{145\pi}{180} \rightarrow \boxed{\frac{29\pi}{36}}$

38.) $-80°$

$-80°\left(\frac{\pi}{180°}\right)$

$\frac{-80\pi}{180} \rightarrow \boxed{-\frac{4\pi}{9}}$

39.) $\frac{4\pi}{9}$

$\frac{4(180°)}{9}$

$\boxed{80°}$

40.) $-\frac{17\pi}{10}$

$\frac{-17(180°)}{10}$

$\boxed{-306°}$

Use your calculator to evaluate the trigonometric functions. Round your answers to the nearest <u>tenth</u>.

41.) $\cot 215°$

$\frac{1}{\tan 215°}$

$\approx \boxed{1.4}$

42.) $\cos \frac{\pi}{8} \rightarrow 22.5°$

$\approx \boxed{0.9}$

43.) $\sec \frac{\pi}{10} \rightarrow 18°$

$\frac{1}{\cos 18°}$

$\approx \boxed{1.1}$

Find the arc length and area of a sector with the given radius *r* and central angle θ. Round answers to the nearest <u>hundredth</u>. Arc length: $s = r\theta$ Area: $A = \frac{1}{2}r^2\theta$ $\theta \rightarrow$ radians!

44.) $r = 5$ ft, $\theta = 90° \rightarrow \frac{\pi}{2}$

$s = 5\left(\frac{\pi}{2}\right)$ $A = \frac{1}{2}(5)^2\left(\frac{\pi}{2}\right)$

$\boxed{s \approx 7.85 \text{ ft}}$ $\boxed{A \approx 19.63 \text{ ft}^2}$

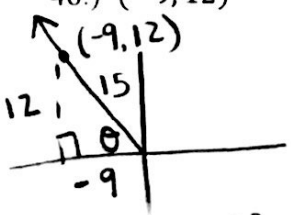
45.) $r = 2$ in., $\theta = 300° \rightarrow \frac{5\pi}{3}$

$s = 2\left(\frac{5\pi}{3}\right)$ $A = \frac{1}{2}(2)^2\left(\frac{5\pi}{3}\right)$

$\boxed{s \approx 10.47 \text{ in}}$ $\boxed{A \approx 10.47 \text{ in}^2}$

Use the given point on the terminal side of an angle θ in standard position to evaluate the six trigonometric functions of θ.

46.) $(-9, 12)$



$(-9)^2 + 12^2 = r^2$

$r = 15$

$\frac{y}{r}$ $\sin\theta = \frac{12}{15} = \frac{4}{5}$

$\frac{x}{r}$ $\cos\theta = \frac{-9}{15} = \frac{-3}{5}$

$\frac{y}{x}$ $\tan\theta = \frac{12}{-9} = \frac{4}{-3}$

$\csc\theta = \frac{5}{4}$

$\sec\theta = \frac{-5}{3}$

$\cot\theta = -\frac{3}{4}$

47.) You and a friend are driving golf balls at a driving range. Your drive has an angle of elevation of 37° with an initial velocity of 140 feet per second. Your friend's drive has an angle of elevation of 45° and an initial velocity of 135 feet per second. Which ball travels the farthest and by how much?

$d = \frac{v^2}{32} \cdot \sin 2\theta$

Me:

$d = \frac{140^2}{32} \cdot \sin(2 \cdot 37°)$

$d \approx 588.77 ft$

Friend:

$d = \frac{135^2}{32} \cdot \sin(2 \cdot 45°)$

$d \approx 569.53 ft$

I hit the ball about 19.24ft further

Solve the equation for θ.

48.) $\sin \theta = 0.27;\ 90° < \theta < 180°$

$\theta = \sin^{-1}(0.27)$

~~15.7°~~

$\theta \approx 15.7°$

$\theta = 180° - 15.7° = \boxed{164.3°}$

49.) $\tan \theta = 0.42;\ 180° < \theta < 270°$

$\theta = \tan^{-1}(0.42)$

~~22.8°~~

$\theta \approx 22.8°$

$\theta = 180° + 22.8° = \boxed{202.8°}$

50.) $\tan \theta = -2.5;\ 270° < \theta < 360°$

$\theta = \tan^{-1}(-2.5)$

~~68.2°~~

$\theta \approx -68.2°$

$\theta = 360° - 68.2° = \boxed{291.8°}$

51.) $\cos \theta = -0.65;\ 180° < \theta < 270°$

$\theta = \cos^{-1}(-0.65)$

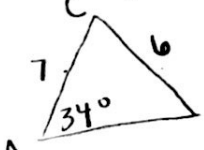
~~49.5°~~

$\theta \approx 130.5°$

$\theta = 180° + 49.5° = \boxed{229.5°}$

Solve ΔABC. Round answers to the nearest tenth.

52.) $A = 34°,\ a = 6,\ b = 7$ SSA *acute!



$\frac{6}{\sin 34°} = \frac{7}{\sin B}$

~~40.7°~~

① $\angle B \approx 40.7°$

$\angle C \approx 105.3°$

$\frac{6}{\sin 34°} = \frac{c}{\sin 105.3°}$

$\boxed{C \approx 10.3}$

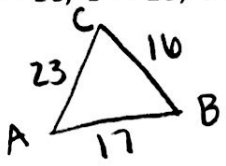
② $\angle B \approx 139.3°$

$\angle C \approx 6.7°$

$\frac{6}{\sin 34°} = \frac{c}{\sin 6.7°}$

$\boxed{C \approx 1.3}$

53.) $a = 16,\ b = 23,\ c = 17$ SSS



$23^2 = 17^2 + 16^2 - 2(17)(16)\cos B$

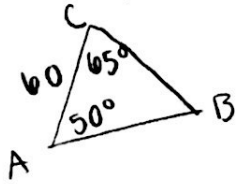
$\boxed{\angle B \approx 88.3°}$

$\frac{23}{\sin 88.3°} = \frac{16}{\sin A}$

$\boxed{\angle A \approx 44.1°}$

$\boxed{\angle C \approx 47.6°}$

54.) $A = 50°,\ C = 65°,\ b = 60$ ASA



$\boxed{\angle B = 65°}$

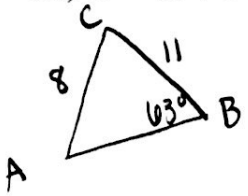
$\frac{60}{\sin 65°} = \frac{c}{\sin 65°}$

$\boxed{C = 60}$

$\frac{60}{\sin 65°} = \frac{a}{\sin 50°}$

$\boxed{a \approx 50.7}$

55.) $B = 63°,\ a = 11,\ b = 8$ SSA *acute!

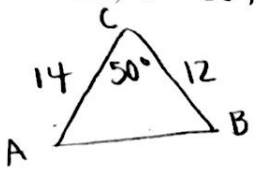


$\frac{8}{\sin 63°} = \frac{11}{\sin A}$

$\angle A \approx X$

$\boxed{\text{NO SOLUTION}}$

56.) $C = 50°,\ a = 12,\ b = 14$ **SAS**



$C^2 = 14^2 + 12^2 - 2(14)(12)\cos 50°$

$C \approx 11.1$ * find $\angle A$ next!

$\frac{11.1}{\sin 50°} = \frac{12}{\sin A}$

$\angle A \approx 55.9°$

$\angle B \approx 74.1°$

57.) $B = 86°,\ b = 13,\ c = 11$ **SSA** * acute!

$\frac{13}{\sin 86°} = \frac{11}{\sin C}$

① $\angle C \approx 57.6°$

$\angle A \approx 36.4°$

$\frac{13}{\sin 86°} = \frac{a}{\sin 36.4°}$

$a \approx 7.7$

~~57.6°~~

② $\angle C \approx 122.4°$ too large! NO second △

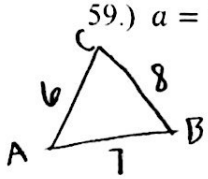
Find the area of ΔABC. Round answers to the nearest tenth.

58.) $A = 81°,\ b = 16,\ c = 18$

$A = \frac{1}{2} \cdot 18 \cdot 16 \cdot \sin 81°$

$A \approx 142.2\ un^2$

59.) $a = 8,\ b = 6,\ c = 7$



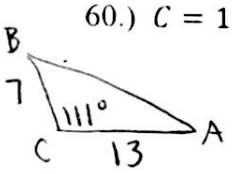
$S = \frac{1}{2}(6+7+8)$

$S = 10.5$

$A = \sqrt{10.5(10.5-8)(10.5-7)(10.5-6)}$

$A \approx 20.3\ un^2$

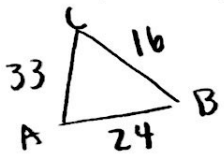
60.) $C = 111°,\ a = 7,\ b = 13$



$A = \frac{1}{2} \cdot 7 \cdot 13 \cdot \sin 111°$

$A \approx 42.5\ un^2$

61.) $a = 16,\ b = 33,\ c = 24$



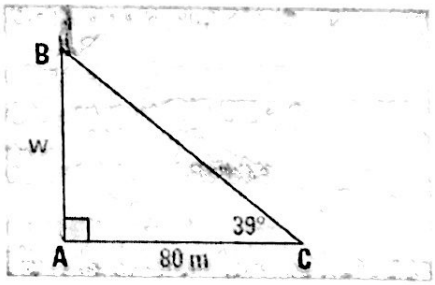
$S = \frac{1}{2}(33+24+16)$

$S = 36.5$

$A = \sqrt{36.5(36.5-33)(36.5-24)(36.5-16)}$

$A \approx 180.9\ un^2$

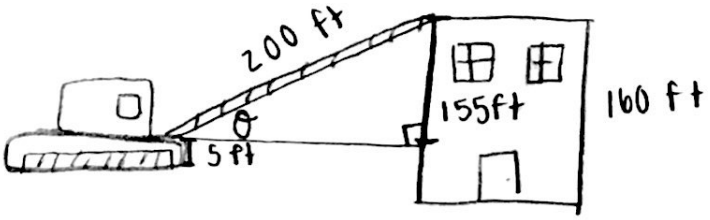
62.) To measure the width of a river, you plant a stake at point A on one side of the riverbank, directly across from a tree stump at point B on the other side of the riverbank. From point A, you walk 80 meters along the riverbank to point C. You find the measure of angle C to be 39°. What is the width w of the river?



$\tan 39° = \frac{w}{80}$

$w \approx 64.8\text{m}$

63.) A crane has a 200 foot arm with a lower end that is 5 feet off the ground. The arm has to reach to the top of a building that is 160 feet high. At what angle θ should the arm be set?



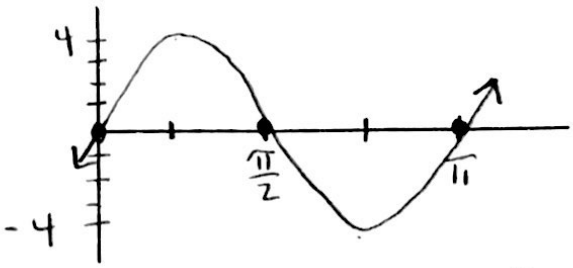
$\sin\theta = \frac{155}{200}$

$\theta \approx 50.8°$

sin & cos period: 2π tan period: π

Graph <u>one period</u> of the function. Identify its domain, range, amplitude, period, and x-/y-intercepts.

64.) $y = 4 \sin 2x$

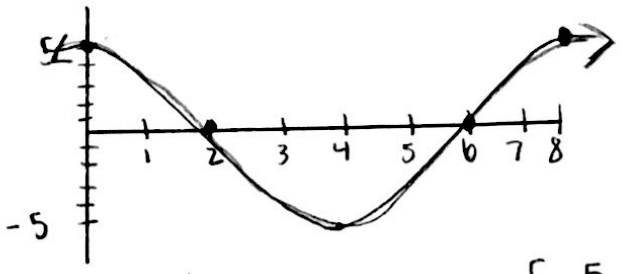


domain: $\mathbb{R}$ range: $[-4, 4]$

amplitude: 4 period: $\frac{2\pi}{2} = \pi$

x-int: $0, \frac{\pi}{2}, \pi$ y-int: $(0, 0)$

65.) $y = 5 \cos \frac{1}{4}\pi x$

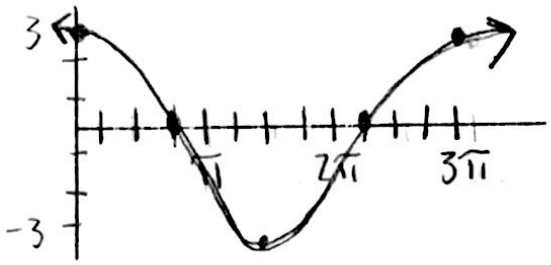


domain: $\mathbb{R}$ range: $[-5, 5]$

amplitude: 5 period: $\frac{2\pi}{\frac{1}{4}\pi} = 8$

x-int: 2, 6 y-int: $(0, 5)$

66.) $y = 3 \cos \frac{2}{3}x$

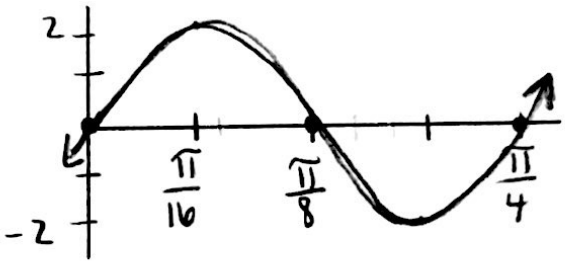


domain: $\mathbb{R}$ range: $[-3, 3]$

amplitude: 3 period: $\frac{2\pi}{\frac{2}{3}} = 3\pi$

x-int: $\frac{3\pi}{4}, \frac{10\pi}{4}$ y-int: $(0, 3)$

67.) $y = 2 \sin 8x$

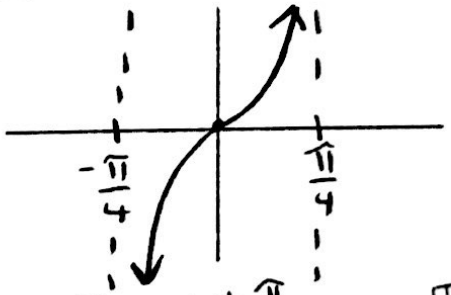


domain: $\mathbb{R}$ range: $[-2, 2]$

amplitude: 2 period: $\frac{2\pi}{8} = \frac{\pi}{4}$

x-int: $0, \frac{\pi}{8}, \frac{\pi}{4}$ y-int: $(0, 0)$

68.) $y = 2 \tan 2x$



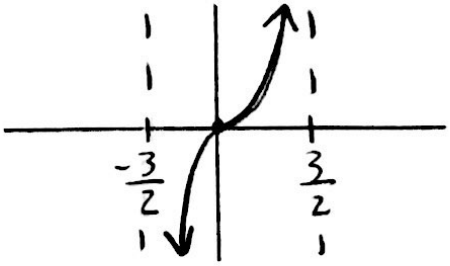
domain: $\mathbb{R}, x \neq \pm\frac{\pi}{4}$ range: $\mathbb{R}$

$\frac{\pi}{2(2)}$ $\frac{\pi}{2}$

asymptotes: $\pm\frac{\pi}{4}$ period: $\frac{\pi}{2}$

x-int: $(0, 0)$ y-int: $(0, 0)$

69.) $y = 3 \tan \frac{1}{3}\pi x$



domain: $\frac{\pi}{2(\frac{1}{3}\pi)}$ range: $\mathbb{R}$

asymptotes: $\pm\frac{3}{2}$ $\frac{\pi}{\frac{1}{3}\pi}$ period: 3

x-int: $(0, 0)$ y-int: $(0, 0)$