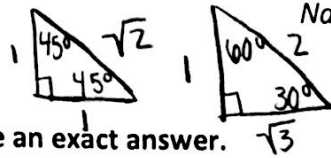


Chapter 13 Review Worksheet

Name: KEY

NO CALCULATOR, NO UNIT CIRCLE



Evaluate the trigonometric function. Give an exact answer.

1.) $\csc \frac{\pi}{6} 30^\circ$
 $\frac{H}{O}$
 $\csc \frac{\pi}{6} = \frac{2}{1} = \boxed{2}$

2.) $\sec \frac{\pi}{3} 60^\circ$
 $\frac{H}{A}$
 $\sec \frac{\pi}{3} = \frac{2}{1} = \boxed{2}$

3.) $\cos \frac{\pi}{4} 45^\circ$
 $\frac{A}{H}$
 $\cos \frac{\pi}{4} = \frac{1}{\sqrt{2}} = \boxed{\frac{\sqrt{2}}{2}}$

4.) $\sin \frac{\pi}{3} 60^\circ$
 $\frac{O}{H}$
 $\sin \frac{\pi}{3} = \frac{\sqrt{3}}{2} = \boxed{\frac{\sqrt{3}}{2}}$

5.) $\cot \frac{\pi}{4} 45^\circ$
 $\frac{A}{O}$
 $\cot \frac{\pi}{4} = \frac{1}{1} = \boxed{1}$

6.) $\tan \frac{\pi}{6} 30^\circ$
 $\frac{O}{A}$
 $\tan \frac{\pi}{6} = \frac{1}{\sqrt{3}} = \boxed{\frac{\sqrt{3}}{3}}$

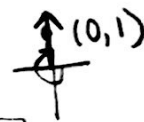
Evaluate the six trigonometric functions of θ .

7.) $\theta = 360^\circ$



$\sin \theta = 0$	$\csc \theta = \frac{1}{0} = \text{UND}$
$\cos \theta = 1$	$\sec \theta = \frac{1}{1} = 1$
$\tan \theta = \frac{0}{1} = 0$	$\cot \theta = \frac{1}{0} = \text{UND}$

8.) $\theta = -\frac{3\pi}{2}$



$\sin \theta = 1$	$\csc \theta = \frac{1}{1} = 1$
$\cos \theta = 0$	$\sec \theta = \frac{1}{0} = \text{UND}$
$\tan \theta = \frac{1}{0} = \text{UND}$	$\cot \theta = \frac{0}{1} = 0$

NO CALCULATOR, MAY USE UNIT CIRCLE

Evaluate the function without using a calculator (i.e. ALL ANSWERS SHOULD BE EXACT, NO DECIMALS).

9.) $\tan 330^\circ$

$\frac{y}{x}$
 $\tan 330^\circ = \frac{-\frac{1}{2}}{\frac{\sqrt{3}}{2}}$
 $= -\frac{1}{2} \cdot \frac{2}{\sqrt{3}} = -\frac{1}{\sqrt{3}} = \boxed{-\frac{\sqrt{3}}{3}}$

10.) $\csc(-405^\circ) \rightarrow 315^\circ$

$\frac{1}{y}$
 $\csc(-405^\circ) = \frac{1}{-\frac{\sqrt{2}}{2}}$
 $= 1 \cdot -\frac{2}{\sqrt{2}} = -\frac{2\sqrt{2}}{2} = \boxed{-\sqrt{2}}$

11.) $\tan 150^\circ$

$\frac{y}{x}$
 $\tan 150^\circ = \frac{\frac{1}{2}}{-\frac{\sqrt{3}}{2}}$
 $= \frac{1}{2} \cdot \frac{2}{-\sqrt{3}} = -\frac{1}{\sqrt{3}} = \boxed{-\frac{\sqrt{3}}{3}}$

12.) $\sec(-480^\circ) \rightarrow 240^\circ$

$\frac{1}{x}$
 $\sec(-480^\circ) = \frac{1}{-\frac{1}{2}}$
 $= 1 \cdot -\frac{2}{1} = \boxed{-2}$

13.) $\sin \frac{13\pi}{6} \rightarrow 30^\circ$

y
 $\sin \frac{13\pi}{6} = \frac{1}{2} = \boxed{\frac{1}{2}}$

14.) $\sec \frac{11\pi}{3} \rightarrow 300^\circ$

$\frac{1}{x}$
 $\sec \frac{11\pi}{3} = \frac{1}{\frac{1}{2}}$
 $= 1 \cdot \frac{2}{1} = \boxed{2}$

Domain \rightarrow $\sin \rightarrow [-90^\circ, 90^\circ]$ $\cos \rightarrow [0^\circ, 180^\circ]$ $\tan \rightarrow (-90^\circ, 90^\circ)$

15.) $\cos^{-1} 1$

$0^\circ, 0 \text{ radians}$

16.) $\tan^{-1} \sqrt{3}$

$60^\circ, \frac{\pi}{3}$

17.) $\sin^{-1} \left(-\frac{\sqrt{2}}{2}\right)$

$-45^\circ, -\frac{\pi}{4}$

18.) $\cos^{-1} \left(-\frac{\sqrt{3}}{2}\right)$

$150^\circ, \frac{5\pi}{6}$

19.) $\sin^{-1} 0$

$0^\circ, 0 \text{ radians}$

20.) $\cos^{-1} 3$

undefined

21.) $\tan^{-1} 1$

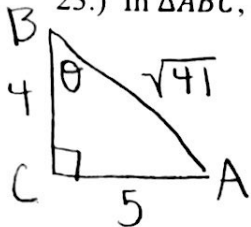
$45^\circ, \frac{\pi}{4}$

22.) $\sin^{-1} \left(-\frac{1}{2}\right)$

$-30^\circ, -\frac{\pi}{6}$

MAY USE CALCULATOR, MAY USE UNIT CIRCLE AND FORMULAS

23.) In $\triangle ABC$, $a = 4, b = 5$, and $C = 90^\circ$. Evaluate the six trigonometric functions of angle B .



$4^2 + 5^2 = c^2$
 $c = \sqrt{41}$

$\frac{O}{H} \sin \theta = \frac{5}{\sqrt{41}} = \frac{5\sqrt{41}}{41}$

$\frac{A}{H} \cos \theta = \frac{4}{\sqrt{41}} = \frac{4\sqrt{41}}{41}$

$\frac{O}{A} \tan \theta = \frac{5}{4}$

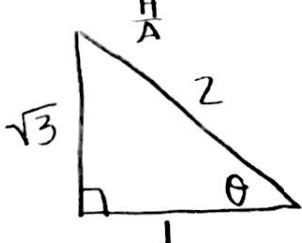
$\csc \theta = \frac{\sqrt{41}}{5}$

$\sec \theta = \frac{\sqrt{41}}{4}$

$\cot \theta = \frac{4}{5}$

Let θ be an acute angle of a right triangle. Find the values of the other five trigonometric functions of θ .

24.) $\sec \theta = 2$



$1^2 + x^2 = 2^2$
 $x = \sqrt{3}$

$\frac{O}{H} \sin \theta = \frac{\sqrt{3}}{2}$

$\frac{A}{H} \cos \theta = \frac{1}{2}$

$\frac{O}{A} \tan \theta = \frac{\sqrt{3}}{1} = \sqrt{3}$

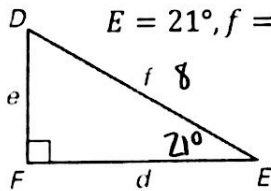
$\csc \theta = \frac{2}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$

$\sec \theta = \frac{2}{1} = 2$

$\cot \theta = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$

Solve $\triangle ABC$ using the diagram and the given measurements. Round answers to the nearest tenth, when necessary.

25.) $E = 21^\circ, f = 8$



$\angle D = 69^\circ$

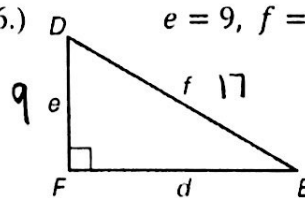
$\sin 21^\circ = \frac{e}{8}$

$e \approx 2.9$

$\cos 21^\circ = \frac{d}{8}$

$d \approx 7.5$

26.) $e = 9, f = 17$



$9^2 + d^2 = 17^2$

$d = \sqrt{208} \approx 14.4$

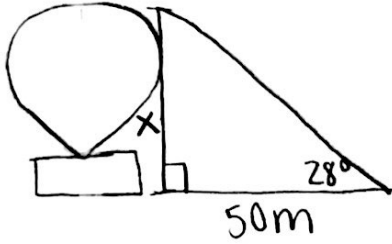
$\cos D = \frac{9}{17}$

$\angle D = \cos^{-1} \left(\frac{9}{17}\right)$

$\angle D \approx 58.0^\circ$

$\angle E \approx 32.0^\circ$

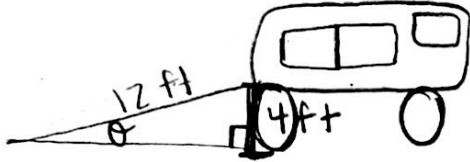
- 27.) You are standing 50 meters from a hot air balloon that is preparing to take off. The angle of elevation to the top of the balloon is 28° . Find the height of the balloon.



$$\tan 28^\circ = \frac{x}{50}$$

$$x \approx 26.6m$$

- 28.) You use a 12 foot ramp to load items into a van. If the floor of the van is 4 feet off the ground, what is the angle of elevation of the ramp?



$$\sin \theta = \frac{4}{12}$$

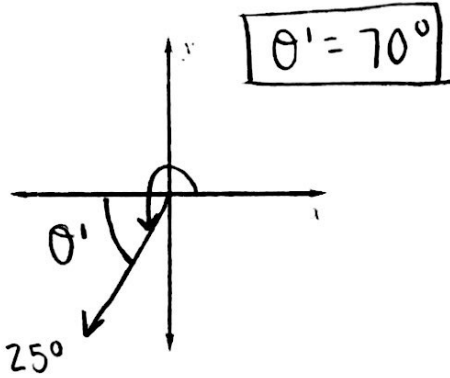
$$\theta = \sin^{-1}\left(\frac{4}{12}\right)$$

$$\theta \approx 19.5^\circ$$

Sketch the angle. Then find its reference angle. Answer in the unit of the given angle.

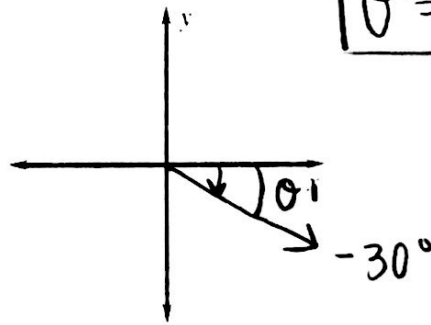
29.) 250°

θ'



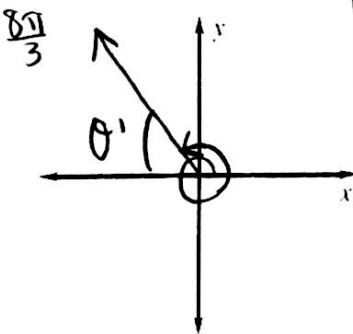
$$\theta' = 70^\circ$$

30.) -30°



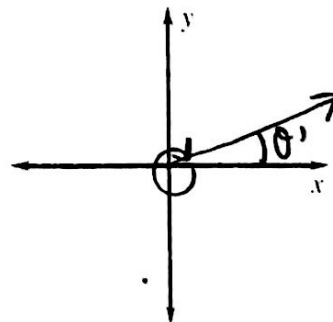
$$\theta' = 30^\circ$$

31.) $\frac{8\pi}{3}$



$$\theta' = \frac{\pi}{3}$$

32.) $-\frac{11\pi}{6}$



$$\theta' = \frac{\pi}{6}$$

$$\pm 360^\circ \text{ or } \pm 2\pi$$

Find one positive angle and one negative angle that are coterminal with the given angle.

33.) $155^\circ \pm 360^\circ$

34.) $-325^\circ \pm 360^\circ$

35.) $\frac{11\pi}{5} \pm \frac{10\pi}{5}$

36.) $\frac{15\pi}{7} \pm \frac{14\pi}{7}$

$$\begin{aligned} + &: 515^\circ, \dots \\ - &: -205^\circ, \dots \end{aligned}$$

$$\begin{aligned} + &: 35^\circ, \dots \\ - &: -685^\circ, \dots \end{aligned}$$

$$\begin{aligned} + &: \frac{\pi}{5}, \frac{21\pi}{5}, \dots \\ - &: -\frac{9\pi}{5}, \dots \end{aligned}$$

$$\begin{aligned} + &: \frac{\pi}{7}, \frac{29\pi}{7}, \dots \\ - &: -\frac{13\pi}{7}, \dots \end{aligned}$$

more possibilities!

Convert the degree measure to radians or the radian measure to degrees.

37.) 145°

38.) -80°

39.) $\frac{4\pi}{9}$

40.) $-\frac{17\pi}{10}$

$$145^\circ \left(\frac{\pi}{180^\circ} \right)$$

$$-80^\circ \left(\frac{\pi}{180^\circ} \right)$$

$$4(180^\circ)$$

$$-17(180^\circ)$$

$$\frac{145\pi}{180} \rightarrow \boxed{\frac{29\pi}{36}}$$

$$\frac{-80\pi}{180} \rightarrow \boxed{-\frac{4\pi}{9}}$$

$$\boxed{80^\circ}$$

$$\frac{-17(180^\circ)}{10} \rightarrow \boxed{-306^\circ}$$

Use your calculator to evaluate the trigonometric functions. Round your answers to the nearest tenth.

41.) $\cot 215^\circ$

42.) $\cos \frac{\pi}{8} \rightarrow 22.5^\circ$

43.) $\sec \frac{\pi}{10} \rightarrow 18^\circ$

$$\frac{1}{\tan 215^\circ} \approx \boxed{1.4}$$

$$\approx \boxed{0.9}$$

$$\frac{1}{\cos 18^\circ} \approx \boxed{1.1}$$

Find the arc length and area of a sector with the given radius r and central angle θ . Round answers to the nearest hundredth. Arc length: $s = r\theta$ Area: $A = \frac{1}{2}r^2\theta$ $\theta \rightarrow$ radians!

44.) $r = 5 \text{ ft}, \theta = 90^\circ \rightarrow \frac{\pi}{2}$

45.) $r = 2 \text{ in.}, \theta = 300^\circ \rightarrow \frac{5\pi}{3}$

$$s = 5\left(\frac{\pi}{2}\right) \quad A = \frac{1}{2}(5)^2\left(\frac{\pi}{2}\right)$$

$$s = 2\left(\frac{5\pi}{3}\right)$$

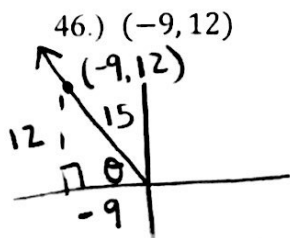
$$A = \frac{1}{2}(2)^2\left(\frac{5\pi}{3}\right)$$

$$\boxed{s \approx 7.85 \text{ ft}} \quad \boxed{A \approx 19.63 \text{ ft}^2}$$

$$\boxed{s \approx 10.47 \text{ in}}$$

$$\boxed{A \approx 10.47 \text{ in}^2}$$

Use the given point on the terminal side of an angle θ in standard position to evaluate the six trigonometric functions of θ .



$$(-9)^2 + 12^2 = r^2$$

$$r = 15$$

$$\frac{y}{r} \quad \sin \theta = \frac{12}{15} = \frac{4}{5}$$

$$\csc \theta = \frac{5}{4}$$

$$\frac{x}{r} \quad \cos \theta = \frac{-9}{15} = -\frac{3}{5}$$

$$\sec \theta = -\frac{5}{3}$$

$$\frac{y}{x} \quad \tan \theta = \frac{12}{-9} = -\frac{4}{3}$$

$$\cot \theta = -\frac{3}{4}$$

47.) You and a friend are driving golf balls at a driving range. Your drive has an angle of elevation of 37° with an initial velocity of 140 feet per second. Your friend's drive has an angle of elevation of 45° and an initial velocity of 135 feet per second. Which ball travels the farthest and by how much?

Me:
 $d = \frac{140^2}{32} \cdot \sin(2 \cdot 37^\circ)$
 $d \approx 588.77 \text{ ft}$

$$d = \frac{v^2}{32} \cdot \sin 2\theta$$

Friend:
 $d = \frac{135^2}{32} \cdot \sin(2 \cdot 45^\circ)$
 $d \approx 569.53 \text{ ft}$

I hit the ball about 19.24 ft further

Solve the equation for θ .

48.) $\sin \theta = 0.27; 90^\circ < \theta < 180^\circ$
 $\theta = \sin^{-1}(0.27)$
 $\theta \approx 15.7^\circ$
 $\theta = 180^\circ - 15.7^\circ = \boxed{164.3^\circ}$

49.) $\tan \theta = 0.42; 180^\circ < \theta < 270^\circ$
 $\theta = \tan^{-1}(0.42)$
 $\theta \approx 22.8^\circ$
 $\theta = 180^\circ + 22.8^\circ = \boxed{202.8^\circ}$

50.) $\tan \theta = -2.5; 270^\circ < \theta < 360^\circ$
 $\theta = \tan^{-1}(-2.5)$
 $\theta \approx -68.2^\circ$
 $\theta = 360^\circ - 68.2^\circ = \boxed{291.8^\circ}$

51.) $\cos \theta = -0.65; 180^\circ < \theta < 270^\circ$
 $\theta = \cos^{-1}(-0.65)$
 $\theta \approx 130.5^\circ$
 $\theta = 180^\circ + 49.5^\circ = \boxed{229.5^\circ}$

Solve $\triangle ABC$. Round answers to the nearest tenth.

52.) $A = 34^\circ, a = 6, b = 7$ **SSA** *acute!

53.) $a = 16, b = 23, c = 17$ **SSS**

Triangle diagram for 52: $\angle A = 34^\circ, a = 6, b = 7$

$\frac{6}{\sin 34^\circ} = \frac{7}{\sin B}$
 $\angle B \approx 40.7^\circ$
 $\angle C \approx 105.3^\circ$

$\frac{6}{\sin 34^\circ} = \frac{c}{\sin 105.3^\circ}$
 $c \approx 10.3$

$\frac{6}{\sin 34^\circ} = \frac{c}{\sin 6.7^\circ}$
 $c \approx 1.3$

Triangle diagram for 53: $a = 16, b = 23, c = 17$

$23^2 = 17^2 + 16^2 - 2(17)(16)\cos B$
 $\angle B \approx 88.3^\circ$

$\frac{23}{\sin 88.3^\circ} = \frac{16}{\sin A}$
 $\angle A \approx 44.1^\circ$
 $\angle C \approx 47.6^\circ$

54.) $A = 50^\circ, C = 65^\circ, b = 60$ **ASA**

55.) $B = 63^\circ, a = 11, b = 8$ **SSA** *acute!

Triangle diagram for 54: $\angle A = 50^\circ, \angle C = 65^\circ, b = 60$

$\angle B = 65^\circ$

$\frac{60}{\sin 65^\circ} = \frac{c}{\sin 65^\circ}$
 $c = 60$

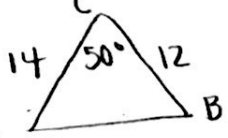
$\frac{60}{\sin 65^\circ} = \frac{a}{\sin 50^\circ}$
 $a \approx 50.7$

Triangle diagram for 55: $\angle B = 63^\circ, a = 11, b = 8$

$\frac{8}{\sin 63^\circ} = \frac{11}{\sin A}$
 $\angle A \approx X$

NO SOLUTION

56.) $C = 50^\circ, a = 12, b = 14$ **SAS**



$$c^2 = 14^2 + 12^2 - 2(14)(12)\cos 50^\circ$$

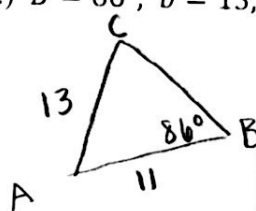
$$c \approx 11.1 \quad \text{* find } \angle A \text{ next!}$$

$$\frac{11.1}{\sin 50^\circ} = \frac{12}{\sin A}$$

$$\angle A \approx 55.9^\circ$$

$$\angle B \approx 74.1^\circ$$

57.) $B = 86^\circ, b = 13, c = 11$ **SSA** *acute!



$$\frac{13}{\sin 86^\circ} = \frac{11}{\sin C}$$

$$\textcircled{1} \angle C \approx 57.6^\circ$$

$$\angle A \approx 36.4^\circ$$

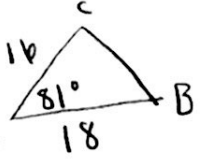
$\textcircled{2} \angle C = 122.4^\circ$
too large!
No second Δ

$$\frac{13}{\sin 86^\circ} = \frac{a}{\sin 36.4^\circ}$$

$$a \approx 7.7$$

Find the area of ΔABC . Round answers to the nearest tenth.

58.) $A = 81^\circ, b = 16, c = 18$



$$A = \frac{1}{2} \cdot 18 \cdot 16 \cdot \sin 81^\circ$$

$$A \approx 142.2 \text{ m}^2$$

59.) $a = 8, b = 6, c = 7$



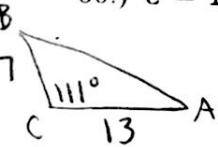
$$A = \sqrt{10.5(10.5-8)(10.5-7)(10.5-6)}$$

$$S = \frac{1}{2}(6+7+8)$$

$$S = 10.5$$

$$A \approx 20.3 \text{ m}^2$$

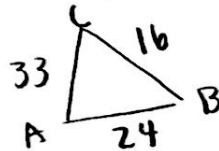
60.) $C = 111^\circ, a = 7, b = 13$



$$A = \frac{1}{2} \cdot 7 \cdot 13 \cdot \sin 111^\circ$$

$$A \approx 42.5 \text{ m}^2$$

61.) $a = 16, b = 33, c = 24$



$$A = \sqrt{36.5(36.5-16)(36.5-33)(36.5-24)}$$

$$S = \frac{1}{2}(33+24+16)$$

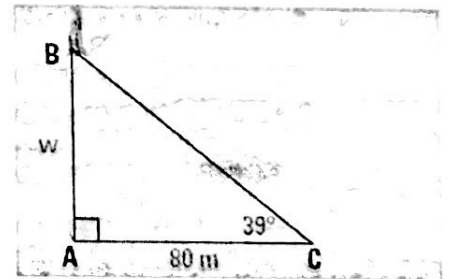
$$S = 36.5$$

$$A \approx 180.9 \text{ m}^2$$

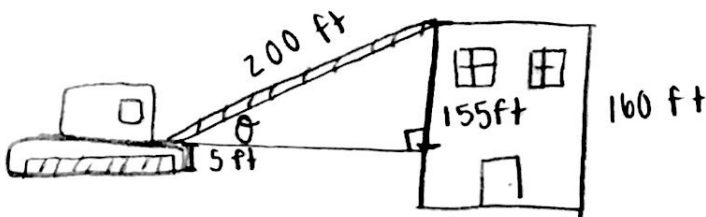
62.) To measure the width of a river, you plant a stake at point A on one side of the riverbank, directly across from a tree stump at point B on the other side of the riverbank. From point A, you walk 80 meters along the riverbank to point C. You find the measure of angle C to be 39° . What is the width w of the river?

$$\tan 39^\circ = \frac{w}{80}$$

$$w \approx 64.8 \text{ m}$$



63.) A crane has a 200 foot arm with a lower end that is 5 feet off the ground. The arm has to reach to the top of a building that is 160 feet high. At what angle θ should the arm be set?



$$\sin \theta = \frac{155}{200}$$

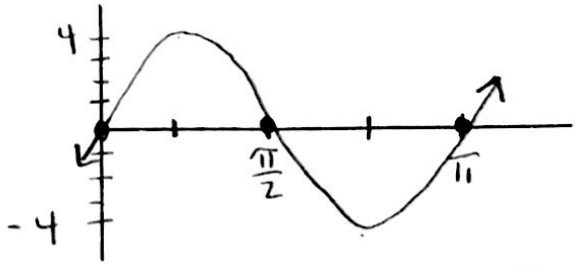
$$\theta \approx 50.8^\circ$$

sin & cos period: 2π

tan period: π

Graph one period of the function. Identify its domain, range, amplitude, period, and x-/y-intercepts.

64.) $y = 4 \sin 2x$

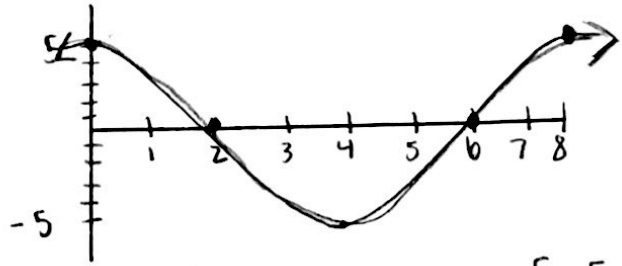


domain: \mathbb{R} range: $[-4, 4]$

amplitude: 4 period: $\frac{2\pi}{2} = \pi$

x-int: $0, \frac{\pi}{2}, \pi$ y-int: $(0, 0)$

65.) $y = 5 \cos \frac{1}{4}\pi x$

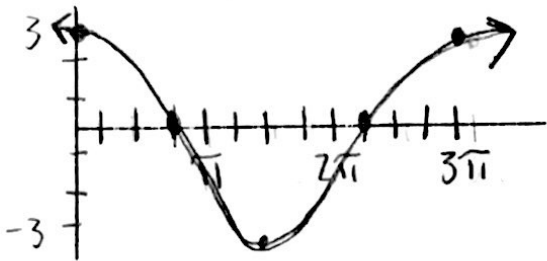


domain: \mathbb{R} range: $[-5, 5]$

amplitude: 5 period: $\frac{2\pi}{\frac{1}{4}\pi} = 8$

x-int: 2, 6 y-int: $(0, 5)$

66.) $y = 3 \cos \frac{2}{3}x$

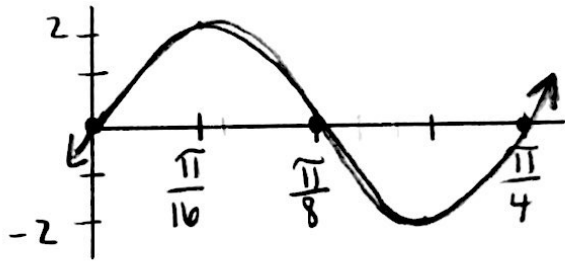


domain: \mathbb{R} range: $[-3, 3]$

amplitude: 3 period: $\frac{2\pi}{\frac{2}{3}} = 3\pi$

x-int: $\frac{3\pi}{4}, \frac{10\pi}{4}$ y-int: $(0, 3)$

67.) $y = 2 \sin 8x$

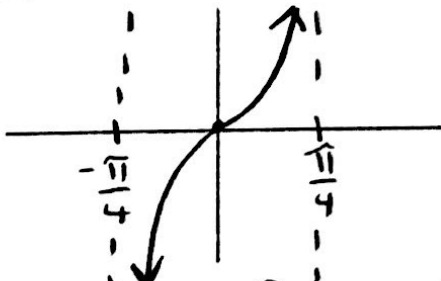


domain: \mathbb{R} range: $[-2, 2]$

amplitude: 2 period: $\frac{2\pi}{8} = \frac{\pi}{4}$

x-int: $0, \frac{\pi}{8}, \frac{\pi}{4}$ y-int: $(0, 0)$

68.) $y = 2 \tan 2x$

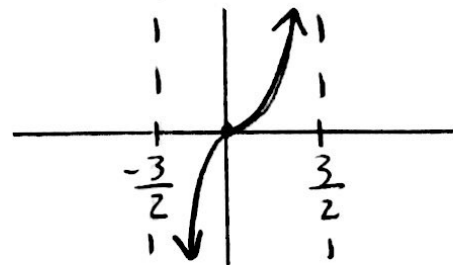


domain: $\mathbb{R}, x \neq \pm \frac{\pi}{4}$ range: \mathbb{R}

asymptotes: $\pm \frac{\pi}{4}$ period: $\frac{\pi}{2}$

x-int: $(0, 0)$ y-int: $(0, 0)$

69.) $y = 3 \tan \frac{1}{3}\pi x$



domain: \mathbb{R} range: \mathbb{R}

asymptotes: $\pm \frac{3}{2}$ period: $\frac{\pi}{\frac{1}{3}\pi} = 3$

x-int: $(0, 0)$ y-int: $(0, 0)$