

NOTES: Section 6.6 – Solve Radical Equations

Goals: #1 - I can solve radical equations and check for extraneous solutions.

#2 - I can solve an equation with two radicals.

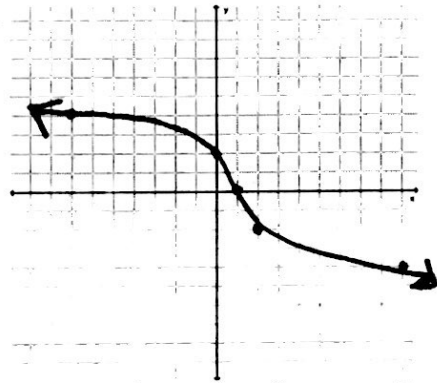


Homework: Lesson 6.6 Worksheet

Warm Up: Graph the function. Then state the domain and range. Lastly, compare the function with its parent function.

1.) $y = -2\sqrt[3]{x-1}$

2.) $y = 2\sqrt{x+3} + 2$

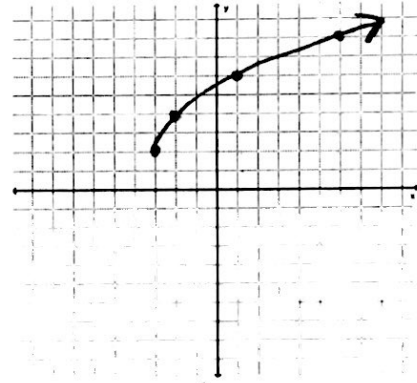


X	y
-7	-4
0	2
1	0
2	-2
9	-4

domain: $(-\infty, \infty)$ or \mathbb{R}
 range: $(-\infty, \infty)$ or \mathbb{R}

comparison:

- reflection over x-axis
- right 1
- vertical stretch



X	y
-3	2
-2	4
1	6
6	8

domain: $[-3, \infty)$
 range: $[2, \infty)$

comparison:

- vertical stretch
- left 3
- up 2

Notes:

To solve a radical equation:

1. Isolate the radical on one side of the equation.
2. Raise each side of the equation to the same power to eliminate radical.
3. Solve the polynomial equation using techniques we've learned.
4. Check your solution!

Example #1: Solve the equation. Check your solution.

1. $\sqrt[3]{x-5} - 1 = -1$
 $\quad +1 \quad +1$

$\sqrt[3]{x-5} = 0$

$(\sqrt[3]{x-5})^3 = (0)^3$

$x-5 = 0$

$+5 \quad +5$

$x = 5$

$\sqrt[3]{5-5} - 1 \stackrel{?}{=} -1$

$\sqrt[3]{0} - 1 \stackrel{?}{=} -1$

$-1 = -1 \checkmark$

3. $x-2 = \sqrt{x+10}$

$(x-2)^2 = (\sqrt{x+10})^2$

$x^2 - 4x + 4 = x + 10$
 $-x \quad -10 \quad -x \quad -10$

$x^2 - 5x - 6 = 0$

$(x-6)(x+1) = 0$

$x = 6$

~~$x = -1$~~

extraneous solution

$6-2 \stackrel{?}{=} \sqrt{6+10}$

$4 \stackrel{?}{=} \sqrt{16}$

$4 = 4 \checkmark$

$-1-2 = \sqrt{-1+10}$

$-3 \neq \sqrt{9}$

$\sqrt{3+6} + 2 = \sqrt{10-6}$

$\sqrt{9} + 2 = \sqrt{4}$

$3+2 = 2$

$5 \neq 2$

extraneous solution

$\sqrt{x+6} = -x$

$(\sqrt{x+6})^2 = (-x)^2$

$x+6 = x^2$

$x^2 - x - 6 = 0$

$(x-3)(x+2) = 0$

~~$x = 3$~~

$x = -2$

2. $(3x+4)^{2/3} = 16$

$\sqrt[3]{(3x+4)^2} = 16$

$(\sqrt[3]{(3x+4)^2})^3 = (16)^3$

$(3x+4)^2 = 4096$

$\sqrt{(3x+4)^2} = \sqrt{4096}$

$3x+4 = 64$

$-4 \quad -4$

$3x = 60$

$x = 20$

$(3(20)+4)^{2/3} \stackrel{?}{=} 16$

$(60+4)^{2/3} \stackrel{?}{=} 16$

$64^{2/3} \stackrel{?}{=} 16$

$(\sqrt[3]{64})^2 \stackrel{?}{=} 16$

$4^2 \stackrel{?}{=} 16$

$16 = 16 \checkmark$

4. $\sqrt{x+6} + 2 = \sqrt{10-3x}$

$(\sqrt{x+6} + 2)^2 = (\sqrt{10-3x})^2$

$(\sqrt{x+6} + 2)(\sqrt{x+6} + 2) = 10-3x$

$\sqrt{x+6} + 2 = \sqrt{10-3x}$
 $x+6 + 2\sqrt{x+6} + 2\sqrt{x+6} + 4 = 10-3x$

$\sqrt{4} + 2 = \sqrt{16}$
 $x+10 + 4\sqrt{x+6} = 10-3x$
 $-x \quad -10 \quad -10 \quad -x$

$2+2 = 4$

$4 = 4 \checkmark$

$4\sqrt{x+6} = -4x$

$\sqrt{x+6} = -x$

$(\sqrt{x+6})^2 = (-x)^2$

$x+6 = x^2$

$x^2 - x - 6 = 0$

$(x-3)(x+2) = 0$

~~$x = 3$~~

$x = -2$

You practice: Solve the equation. Check your solution.

1. $-2x^{4/3} - 21 = -53$

$$\begin{array}{r} -2x^{4/3} - 21 = -53 \\ +21 \quad +21 \\ \hline -2x^{4/3} = -32 \\ \hline \frac{-2x^{4/3}}{-2} = \frac{-32}{-2} \end{array}$$

$$\begin{array}{l} x^{4/3} = 16 \\ (\sqrt[3]{x})^4 = 16 \\ \sqrt[4]{(\sqrt[3]{x})^4} = \sqrt[4]{16} \end{array}$$

$$\begin{array}{l} \sqrt[3]{x} = \pm 2 \\ (\sqrt[3]{x})^3 = (\pm 2)^3 \\ \boxed{x = \pm 8} \end{array}$$

$$\begin{array}{l} -2(8)^{4/3} - 21 \stackrel{?}{=} -53 \\ -2(16) - 21 \stackrel{?}{=} -53 \\ -53 = -53 \checkmark \\ -2(-8)^{4/3} - 21 \stackrel{?}{=} -53 \\ -2(16) - 21 \stackrel{?}{=} -53 \\ -53 = -53 \checkmark \end{array}$$

2. $x + 2 = \sqrt{2x + 7}$

$$\begin{array}{l} (x+2)^2 = (\sqrt{2x+7})^2 \\ x^2 + 4x + 4 = 2x + 7 \\ -2x \quad -7 \quad -2x \quad -7 \end{array}$$

$$x^2 + 2x - 3 = 0$$

$$(x+3)(x-1) = 0$$

extraneous solution $\boxed{x = 3}$

$$\begin{array}{l} 3 + 2 \stackrel{?}{=} \sqrt{2(-3) + 7} \\ 5 \stackrel{?}{=} \sqrt{-6 + 7} \\ 5 \neq 1 \end{array}$$

$$\begin{array}{l} \boxed{x = 1} \\ 1 + 2 \stackrel{?}{=} \sqrt{2(1) + 7} \\ 3 = \sqrt{9} \\ 3 = 3 \checkmark \end{array}$$

3. $\sqrt{3x+4} - 1 = \sqrt{x+5}$

$$(\sqrt{3x+4} - 1)^2 = (\sqrt{x+5})^2$$

$$(\sqrt{3x+4} - 1)(\sqrt{3x+4} - 1) = x + 5$$

$$3x + 4 - 2\sqrt{3x+4} + 1 = x + 5$$

$$\begin{array}{r} 3x + 5 - 2\sqrt{3x+4} = x + 5 \\ -3x \quad -5 \quad -3x \quad -5 \end{array}$$

$$\begin{array}{r} -2\sqrt{3x+4} = -2x \\ \hline \frac{-2\sqrt{3x+4}}{-2} = \frac{-2x}{-2} \end{array}$$

$$\sqrt{3x+4} = x$$

$$(\sqrt{3x+4})^2 = (x)^2$$

$$3x + 4 = x^2$$

$$x^2 - 3x - 4 = 0$$

$$(x-4)(x+1) = 0$$

$$\boxed{x = 4} \quad \boxed{x = -1}$$

$$\begin{array}{l} \sqrt{3(4)+4} - 1 \stackrel{?}{=} \sqrt{4+5} \\ \sqrt{12+4} - 1 \stackrel{?}{=} \sqrt{9} \\ \sqrt{16} - 1 \stackrel{?}{=} 3 \\ 4 - 1 \stackrel{?}{=} 3 \\ 3 = 3 \checkmark \end{array}$$

extraneous solution

$$\begin{array}{l} \sqrt{3(-1)+4} - 1 \stackrel{?}{=} \sqrt{-1+5} \\ \sqrt{-3+4} - 1 \stackrel{?}{=} \sqrt{4} \\ \sqrt{1} - 1 \stackrel{?}{=} 2 \\ 1 - 1 \stackrel{?}{=} 2 \\ 0 \neq 2 \end{array}$$