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NOTES: Section 7.6 – Solve Exponential and Logarithmic Equations

Goals: #1 - I can solve an exponential equation by rewriting both sides with a common base.

#2 - I can solve an exponential equation by taking a logarithm of both sides.

#3 - I can solve a logarithmic equation by canceling out logarithms.

#4 - I can solve a logarithmic equation by using exponents.

Homework: Lesson 7.6 Worksheet

Warm Up:

1. Expand the expression.

a. $\log_3 15x$

$\boxed{\log_3 15 + \log_3 x}$

b. $\ln \frac{\sqrt[3]{x}}{y^2}$

$\ln \sqrt[3]{x} - \ln y^2$

$\ln x^{1/3} - \ln y^2$

$\boxed{\frac{1}{3} \ln x - 2 \ln y}$

2. Condense the expression.

a. $5 \log_2 x - 4 \log_2 y$

$\log_2 x^5 - \log_2 y^4$

$\boxed{\log_2 \frac{x^5}{y^4}}$

b. $\ln 4 + 3 \ln 3 - \ln 12$

$\ln 4 + \ln 3^3 - \ln 12$

$\ln 4 + \ln 27 - \ln 12$

$\ln 4 \cdot 27 - \ln 12$

$\ln 108 - \ln 12$

$\ln \frac{108}{12} \rightarrow \boxed{\ln 9}$

Notes:

Exponential equations are equations in which the variable occurs in the exponent.

Example: $2^x = 4$ $e^x - 3 = 7$

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Example #1: Solve the exponential equation.

1. $4^x = \left(\frac{1}{2}\right)^{x-3}$

$(2^2)^x \quad (2^{-1})^{x-3}$

same base $\quad 2^{2x} = 2^{-x+3}$

$2x = -x+3$

$3x = 3$

$\boxed{x=1}$

2. $100^{7x+1} = 1000^{3x-2}$

$(10^2)^{7x+1} = (10^3)^{3x-2}$

same base $\quad 10^{14x+2} = 10^{9x-6}$

$14x+2 = 9x-6$

$5x =$

$\boxed{x = \ }$

You practice: Solve the exponential equation.

1. $9^{2x} = 27^{x-1}$

$(3^2)^{2x} = (3^3)^{x-1}$

$3^{4x} = 3^{3x-3}$

$4x = 3x-3$

$\boxed{x=-3}$

2. $81^{3-x} = \left(\frac{1}{3}\right)^{5x-6}$

$(3^4)^{3-x} = (3^{-1})^{5x-6}$

$3^{12-4x} = 3^{-5x+6}$

$12-4x = -5x+6$

$\boxed{x=-6}$

Notes:

How would we solve the equation $4^x = 11$?

We <u>cannot</u> write each side with the <u>same</u> base.

To solve these types of <u>exponential</u> equations, we will use <u>logs</u>.

Example #2: Solve the exponential equation.

1. $4^x = 11$

$\cancel{\log_4 4}^x = \log_4 11$

$x = \log_4 11$

$x = \frac{\log 11}{\log 4}$

$\boxed{x \approx 1.73}$

2. $4e^{-0.3x} - 7 = 13$

$4e^{-0.3x} = 20$

$e^{-0.3x} = 5$

$\cancel{\ln e}^{-0.3x} = \ln 5$

$-0.3x = \ln 5$

$\boxed{x \approx -5.365}$

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You practice: Solve the exponential equation.

1. $2^x = 5$

$\cancel{\log_2 2}^x = \log_2 5$

$x = \log_2 5$

$x = \frac{\log 5}{\log 2}$

$\boxed{x \approx 2.322}$

2. $10^{3x} + 4 = 9$

$10^{3x} = 5$

$\cancel{\log 10}^{3x} = \log 5$

$3x = \log 5$

$\boxed{x \approx 0.233}$

Notes:

Logarithmic equations are equations in which the variable occurs in the logarithm.

Example: $\log_5 x = 7$ $\ln(x-2) = \ln 3x$

Example #3: Solve the logarithmic equation.

1. $\log_5(4x - 7) = \log_5(x + 4)$

same base

$4x - 7 = x + 4$

$3x = 11$

$\boxed{x = 11/3}$

check:
$(4(11/3) - 7)$
(7.7) ✓
$(11/3 + 4)$ ✓

2. $\ln(7x - 4) = \ln(2x + 11)$

same base

$7x - 4 = 2x + 11$

$5x = 15$

$\boxed{x = 3}$

check:
$(7(3) - 4)$
(17) ✓
$(2(3) + 11)$
(17) ✓

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Notes:

How would we solve the equation $\log_4(5x-1)=3$?

We <u>cannot</u> write each side with the <u>same</u> logarithmic base.

To solve these types of <u>logarithmic</u> equations, we will use <u>exponents</u>

Example #4: Solve the logarithmic equation.

1. $\log_4(5x-1)=3$

$\cancel{4^{\log_4}}(5x-1)=4^3$

$5x-1=64$

$5x=65$

$\boxed{x=13}$

Check:
(5(13)-1)
(64) ✓

2. $\log 5x+\log(x-1)=2$

$\log(5x(x-1))=2$

$\log(5x^2-5x)=2$

$\cancel{10^{\log}}(5x^2-5x)=10^2$

$5x^2-5x=100$

$5x^2-5x-100=0$

$5(x^2-x-20)=0$

$(x-5)(x+4)=0$

$\boxed{x=5}$ $\boxed{\cancel{x=-4}}$ extraneous

check
(5-1)
(4) ✓
(-4-1)
($\cancel{-5}$)

You practice: Solve the logarithmic equation.

1. $\log_2(x-6)=5$

$\cancel{2^{\log_2}}(x-6)=2^5$

$x-6=32$

$\boxed{x=38}$

Check:
(38-6)
(32) ✓

2. $\log_4(x+12)+\log_4 x=3$

$\log_4(x(x+12))=3$

$\log_4(x^2+12x)=3$

$\cancel{4^{\log_4}}(x^2+12x)=4^3$

$x^2+12x=64$

$x^2+12x-64=0$

$(x+16)(x-4)=0$

$\boxed{\cancel{x=-16}}$ $\boxed{x=4}$

extraneous

check:
(4) ✓
($\cancel{-16}$)

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Example #5: You deposit $100 in an account that pays 6% annual interest compounded daily. How long will it take for the balance to reach $1000?

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$

$$1000 = 100\left(1 + \frac{0.06}{365}\right)^{365t}$$

$$1000 = 100(1.0001\ldots)^{365t}$$

$$10 = (1.0001\ldots)^{365t}$$

$$\log_{1.0001\ldots} 10 = \cancel{\log_{1.0001\ldots}(1.0001\ldots)}^{365t}$$

$$\frac{\log 10}{\log(1.0001\ldots)} = 365t$$

$$14008.54391 = 365t$$

$$t \approx \boxed{38.4 \text{ years}}$$