

## NOTES: Section 7.5 – Special Types of Linear Systems

Goals: #1 - I can identify how many solutions a linear system has.

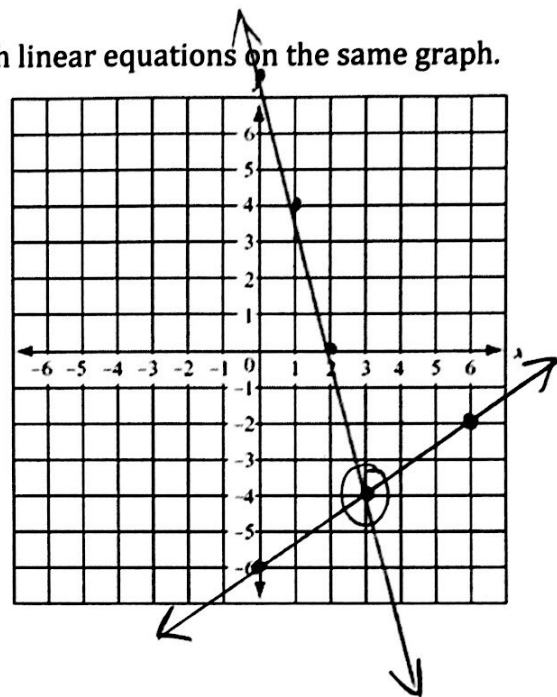


### Homework: Section 7.5 Worksheet

Exploration #1: Work with a partner. Graph both linear equations on the same graph.

$$\begin{aligned}
 4x + y &= 8 \\
 -4x &\quad -4x \\
 \hline
 y &= -4x + 8
 \end{aligned}$$
  

$$\begin{aligned}
 2x - 3y &= 18 \\
 -2x &\quad -2x \\
 \hline
 -3y &= -2x + 18 \\
 \frac{-3y}{-3} &= \frac{-2x}{-3} + \frac{18}{-3} \\
 y &= \frac{2}{3}x - 6
 \end{aligned}$$



Circle where these lines intersect. Can you check if your answer is correct?

$(3, -4)$

$$\begin{aligned}
 4(3) + (-4) &\stackrel{?}{=} 8 & 2(3) - 3(-4) &\stackrel{?}{=} 18 \\
 12 - 4 &\stackrel{?}{=} 8 & 6 + 12 &\stackrel{?}{=} 18 \\
 8 &= 8 \checkmark & 18 &= 18 \checkmark
 \end{aligned}$$

Notes:

A linear system consists of two linear equations.

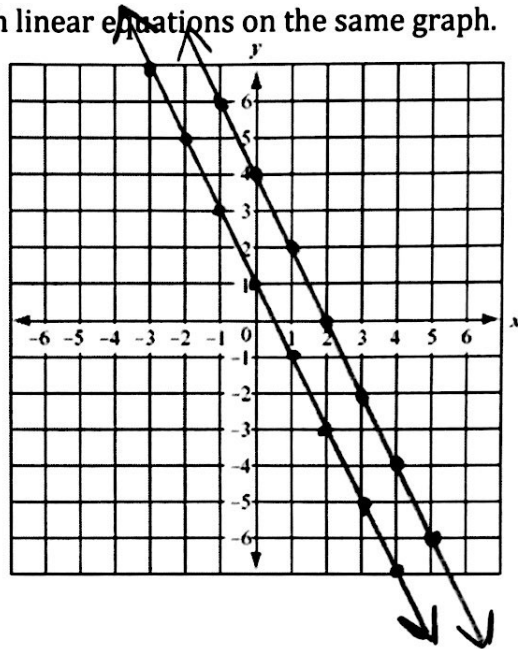
A solution of a system of linear equations, is an ordered pair  $(x, y)$  where the graphs of the equations in a system intersect.

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Exploration #2: Work with a partner. Graph both linear equations on the same graph.

$$\begin{array}{r} 2x + y = 4 \\ -2x \quad -2x \\ \hline y = -2x + 4 \end{array}$$

$$\begin{array}{r} 2x + y = 1 \\ -2x \quad -2x \\ \hline y = -2x + 1 \end{array}$$



Circle where these lines intersect. Can you check if your answer is correct?

They don't ☹️

Notes:

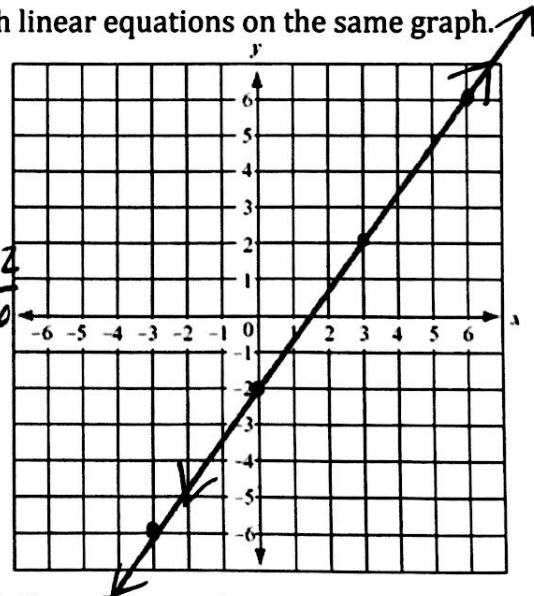
Lines that never intersect are called parallel lines.

Since the graphs of the system do NOT intersect, we have no solution.

Exploration #3: Work with a partner. Graph both linear equations on the same graph.

$$\begin{array}{r} 4x - 3y = 6 \\ -4x \quad -4x \\ \hline -3y = -4x + 6 \\ \frac{-3y}{-3} = \frac{-4x + 6}{-3} \\ y = \frac{4}{3}x - 2 \end{array}$$

$$\begin{array}{r} 8x - 6y = 12 \\ -8x \quad -8x \\ \hline -6y = -8x + 12 \\ \frac{-6y}{-6} = \frac{-8x + 12}{-6} \\ y = \frac{4}{3}x - 2 \end{array}$$



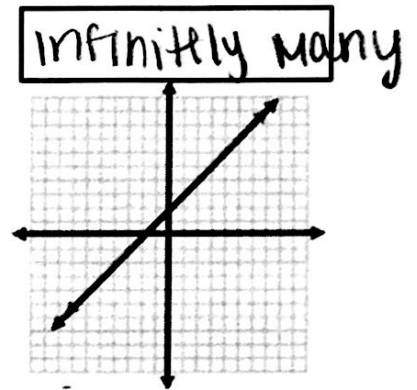
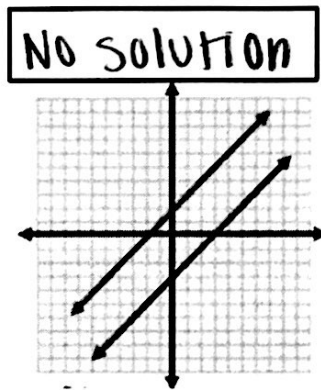
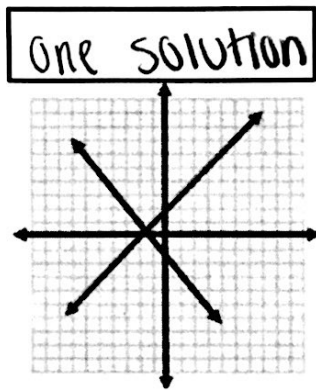
Circle where these lines intersect. Can you check if your answer is correct?

↘ at every point!

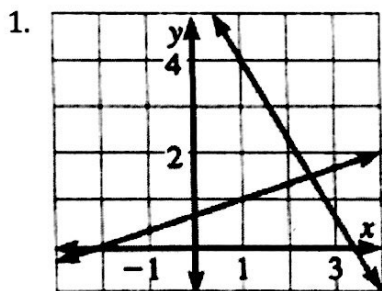
Notes:

Lines that intersect at every point are the same line!

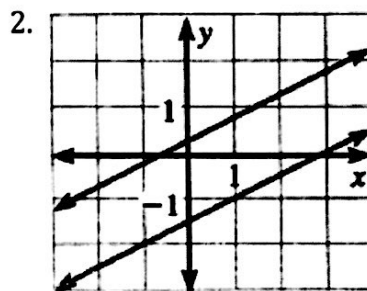
Since the graphs of the system intersect at EVERY point, we have infinitely many solutions.



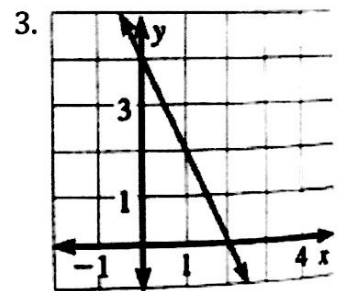
Example #1: Tell how many solutions the system has.



ONE



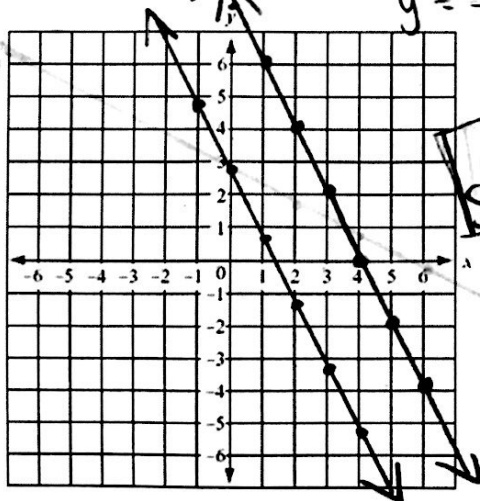
NO SOLUTION



INFINITELY MANY

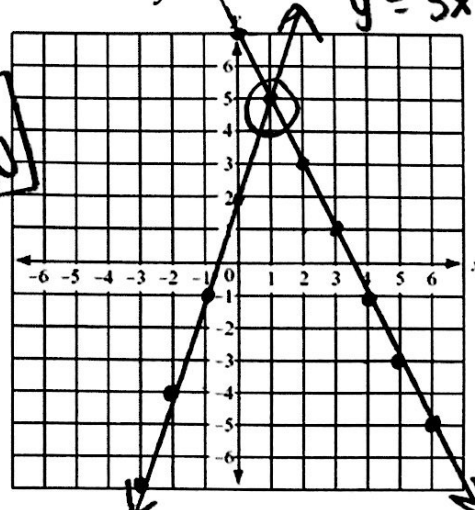
Example #2: Use the graphing method to tell how many solutions the system has.

1.  $2x + y = 8 \rightarrow y = -2x + 8$   
 $-6x - 3y = -8 \rightarrow -3y = 6x - 8$   
 $y = -2x + \frac{8}{3}$



NO SOLUTION

2.  $2x + y = 7 \rightarrow y = -2x + 7$   
 $3x - y = -2 \rightarrow -y = -3x - 2$   
 $y = 3x + 2$

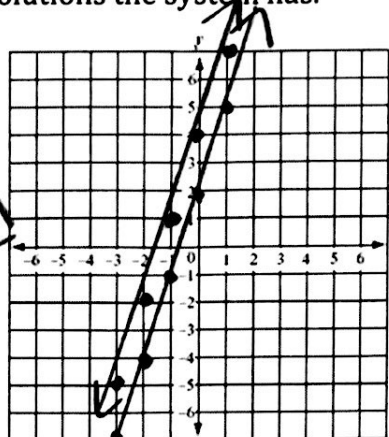


ONE SOLUTION  
(1, 5)

Warm Up: Use the graphing method to tell how many solutions the system has.

$$\begin{aligned}
 1. \quad & -6x + 2y = 4 \\
 & +6x \quad +6x \\
 & \quad 2y = 6x + 4 \\
 & \quad y = 3x + 2 \\
 \\ 
 & -9x + 3y = 12 \\
 & +9x \quad +9x \\
 & \quad 3y = 9x + 12 \\
 & \quad y = 3x + 4
 \end{aligned}$$

NO SOLUTION



2. When would a linear system have infinitely many solutions?  
*when they are the same line!*

Review:

We know that when we solve linear systems, we could have ONE solution, NO solution, or INFINITELY MANY solutions.

What does this look like algebraically?

ONE SOLUTION

$$\begin{aligned}
 x &= 3 \\
 y &= -1 \\
 &(3, -1)
 \end{aligned}$$

NO SOLUTION

when variables drop out

$$12 \neq 8$$

INFINITELY MANY SOLUTIONS

variables drop out

$$0 = 0$$

Example #3: Use substitution or elimination to solve the linear system.

a.  $x - 2y = 4 \rightarrow x = 4 + 2y$

$$3x - 6y = 8$$

$$3(4 + 2y) - 6y = 8$$

$$12 + 6y - 6y = 8$$

$$12 \neq 8$$

NO SOLUTION

b.  $3(3x + y = -1)$

$$\begin{array}{r}
 -9x - 3y = 3 \\
 + \quad 9x + 3y = -3 \\
 \hline
 0 = 0
 \end{array}$$

INFINITELY MANY SOLUTIONS

**Example #4:** Use substitution or elimination to solve the linear system. Then describe the graph of the system.

1.  $-x + y = 7 \Rightarrow y = 7 + x$

$2x - 2y = -18$

$2x - 2(7 + x) = -18$

$2x - 14 - 2x = -18$

$-14 \neq -18$

NO SOLUTION

These lines would be parallel lines.

$-3 \cdot (-4x + y = -8)$

$-12x + 3y = -24$

$+ \quad 12x - 3y = 24$

$0 = 0$

INFINITELY MANY SOLUTIONS

These lines would be the same line.

3.  $-4x + y = -8$

$4(2x - 2y = -14)$

$+ \quad 4x - 4y = -28$   
 $-4x + y = -8$

$3y = -36$

$y = 12$

$-4x + 12 = -8$

$-4x = -20$

$x = 5$

These lines would intersect at  $(5, 12)$

ONE SOLUTION  $(5, 12)$