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# NOTES: Section 3.4 – Solve Systems of Linear Equations in Three Variables

Goals: #1 - I can solve a 3 variable system using elimination (with exactly one, zero, or infinitely many solutions).

#2 - I can solve a 3 variable system using substitution (with exactly one, zero, or infinitely many solutions).

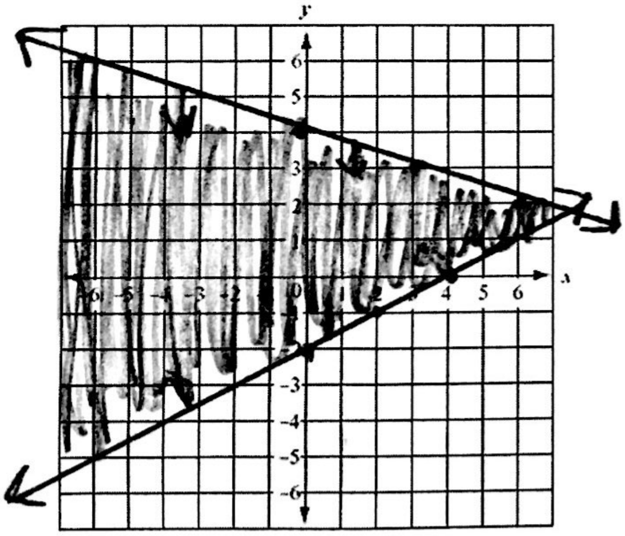
*Homework: Lesson 3.4 Worksheet*

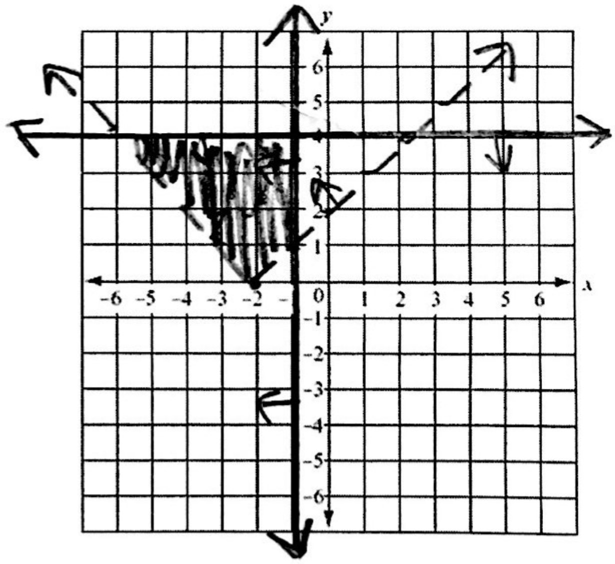
**Warm Up:** Graph the system of inequalities.

1. $2x - 4y \leq 8$
   $y < -\frac{1}{3}x + 4$

   $-4y \leq -2x + 8$
   $y \geq \frac{1}{2}x - 2$

2. $y > |x + 2|$
   $y \leq 4$
   $x \leq -1$





**Notes:**

A three-variable system consists of three equations.

A solution of this system, is an ordered triple $(x, y, z)$ where the planes in the system intersect at exactly one point.

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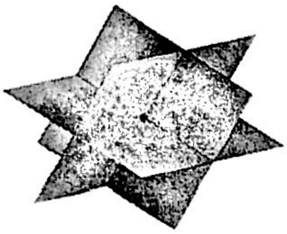
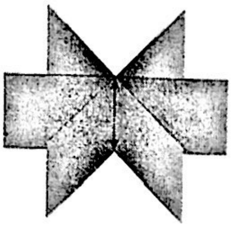
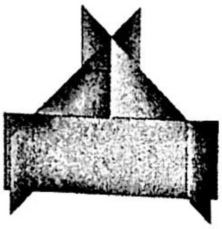
**Exactly one solution**
The planes intersect in a single point.

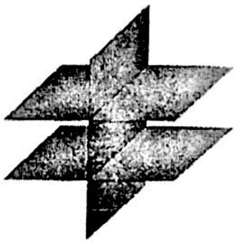


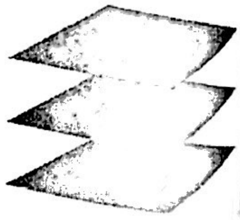
**Infinitely many solutions**
The planes intersect in a line or are the same plane.



**No solution**
The planes have no common point of intersection.







There are two algebraic methods for solving system of three equations:
substitution and elimination

**Example #1:** Solve the system using <u>elimination.</u>

① $4x + 2y + 3z = 1$

② $2x - 3y + 5z = -14$  eliminate y

③ $6x - y + 4z = -1$

**Step 1:** Rewrite each system in *two* variables (eliminate a chosen variable).

① $4x + 2y + 3z = 1$
③ $2(6x - y + 4z = -1)$
$12x - \cancel{2y} + 8z = -2$
$+\ 4x + \cancel{2y} + 3z = 1$
$16x + 11z = -1$

② $2x - 3y + 5z = -14$
③ $-3(6x - y + 4z = -1)$
$-18x + \cancel{3y} - 12z = 3$
$2x - \cancel{3y} + 5z = -14$
$-16x - 7z = -11$

**Step 2:** Solve for both variables.

$+\ \cancel{16x} + 11z = -1$
$-\cancel{16x} - 7z = -11$
$4z = -12$
$\boxed{z = -3}$

$16x + 11(-3) = -1$
$16x - 33 = -1$
$16x = 32$
$\boxed{x = 2}$

**Step 3:** Substitute and solve for the remaining variable.

$6(2) - y + 4(-3) = -1$
$12 - y - 12 = -1$
$-y = -1$
$\boxed{y = 1}$

$\boxed{(2, 1, -3)}$

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**Review:**

We know that when we solve linear systems, we could have ONE solution, NO solution, or infinitely many solutions. This is the same for three-variable systems

What does this look like algebraically?

| ONE SOLUTION | NO SOLUTION | INFINITELY MANY SOLUTIONS |
|---|---|---|
| $\boxed{x = \#}$ $\boxed{y = \#}$ $\boxed{z = \#}$ | all variables drop out $0 \neq -5$ | all variables drop out $0 = 0$ ✓ |

**Example #2:** Solve the system using <u>elimination.</u>

elim. y

① $x + y + z = 3$
② $4x + 4y + 4z = 7$
③ $3x - y + 2z = 5$

$$\begin{aligned} ①\ & x + \cancel{y} + z = 3 \\ +\ ③\ & 3x - \cancel{y} + 2z = 5 \\ \hline & 4x \quad + 3z = 8 \end{aligned}$$

$$\begin{aligned} ②\ & 4x + 4y + 4z = 7 \\ ③\ & 4(3x - y + 2z = 5) \\ & 12x - \cancel{4y} + 8z = 20 \\ +\ & 4x + \cancel{4y} + 4z = 7 \\ \hline & 16x \quad + 12z = 27 \end{aligned}$$

$$\begin{aligned} & 16x + 12z = 27 \\ & -4(4x + 3z = 8) \\ +\ & -16x - 12z = -32 \\ & 16x + 12z = 27 \\ \hline & 0 \neq -5 \end{aligned}$$

$\boxed{\text{no solution}}$

**Example #3:** Solve the system using <u>elimination.</u>

elim. z

① $x + y + z = 4$
② $x + y - z = 4$
③ $3x + 3y + z = 12$

$$\begin{aligned} ①\ & x + y + \cancel{z} = 4 \\ +\ ②\ & x + y - \cancel{z} = 4 \\ \hline & 2x + 2y \quad = 8 \end{aligned}$$

$$\begin{aligned} ②\ & x + y - \cancel{z} = 4 \\ +\ ③\ & 3x + 3y + \cancel{z} = 12 \\ \hline & 4x + 4y \quad = 16 \end{aligned}$$

$$\begin{aligned} & 4x + 4y = 16 \\ & -2(2x + 2y = 8) \\ +\ & -4x - 4y = -16 \\ & 4x + 4y = 16 \\ \hline & 0 = 0\ ✓ \end{aligned}$$

$\boxed{\text{infinitely many solutions}}$

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**Example #4:** Solve the system using <u>substitution</u>.

$$2x + y + z = 8$$
$$-x + 3y - 2z = 3$$
$$y = x + z$$

$2x + (x + z) + z = 8$

$3x + 2z = 8$

$-x + 3(x + z) - 2z = 3$

$-x + 3x + 3z - 2z = 3$

$2x + z = 3$

$z = 3 - 2x$

$3x + 2(3 - 2x) = 8$

$3x + 6 - 4x = 8$

$6 - x = 8$

$-x = 2$

$\boxed{x = -2}$

$z = 3 - 2(-2)$

$z = 3 + 4$

$\boxed{z = 7}$

$\boxed{(-2, 5, 7)}$

$y = x + z$

$y = -2 + 7 \rightarrow \boxed{y = 5}$

**Example #5:** At a carry-out pizza restaurant, an order of 3 slices of pizza, 4 breadsticks, and 2 sodas costs $13.35. A second order of 5 slices of pizza, 2 breadsticks, and 3 sodas costs $19.50. If 4 breadsticks and a soda cost $0.30 more than a slice of pizza, what is the cost of each item?

$3x + 4y + 2z = 13.35$

$5x + 2y + 3z = 19.50$

$4y + z = x + 0.30$

x = cost of pizza

y = cost of breadsticks

z = cost of soda

pizza: $2.95

breadsticks: $0.50

soda: $1.25

$x = 4(0.50) + 1.25 - 0.30$

$\boxed{x = \$2.95}$

$x = 4y + z - 0.30$

$3(4y + z - 0.30) + 4y + 2z = 13.35$

$12y + 3z - 0.90 + 4y + 2z = 13.35$

$16y + 5z - 0.90 = 13.35$

$16y + 5z = 14.25$

$5z = 14.25 - 16y$

$z = 2.85 - 3.2y$

$z = 2.85 - 3.2(0.50)$

$\boxed{z = \$1.25}$

$5(4y + z - 0.30) + 2y + 3z = 19.50$

$20y + 5z - 1.50 + 2y + 3z = 19.50$

$22y + 8z - 1.50 = 19.50$

$22y + 8z = 21$

$22y + 8(2.85 - 3.2y) = 21$

$22y + 22.8 - 25.6y = 21$

$-3.6y = -1.8$

$\boxed{y = \$0.50}$