Name: KEY Hour: ____________ Date: ____________

NOTES: Section 13.6 – Apply the Law of Cosines

Goals: #1 - I can solve a triangle using the Law of Cosines.

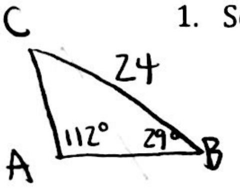
#2 - I can use Heron's area formula to find the area of a triangle when given 3 side lengths.

Homework: Lesson 13.6 Worksheet

Warm Up:

1. Solve ΔABC. Round answers to the nearest <u>tenth</u>.

a. $A = 112°, a = 24, B = 29°$



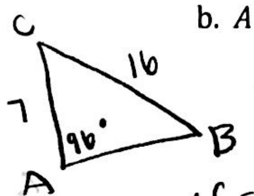
$\angle C = 180° - 112° - 29°$

$\boxed{\angle C = 39°}$

$\frac{24}{\sin 112°} = \frac{b}{\sin 29°}$ $\boxed{b \approx 12.5}$

$\frac{24}{\sin 112°} = \frac{c}{\sin 39°}$ $\boxed{c \approx 16.3}$

b. $A = 96°, a = 16, b = 7$



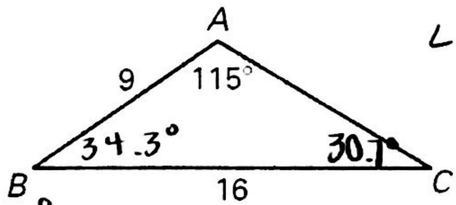
$\frac{16}{\sin 96°} = \frac{7}{\sin B}$ $\boxed{\angle B \approx 25.8}$

$\angle C = 180° - 96° - 25.8°$

$\boxed{\angle C \approx 58.2°}$

$\frac{16}{\sin 96°} = \frac{c}{\sin 58.2°}$ $\boxed{c \approx 13.7}$

2. Find the area of ΔABC.



$\frac{16}{\sin 115°} = \frac{9}{\sin C}$ $\angle C \approx 30.7°$

$\angle B = 180° - 115° - 30.7°$

$\angle B = 34.3°$

$A = \frac{1}{2}(9)(16)\sin 34.3°$

$A \approx \boxed{40.6\ u^2}$

Review:

We can use <u>Law of Sines</u> for triangles in the following cases:

- <u>AAS</u>: 
- <u>ASA</u>: 
- <u>SSA</u>: 
  - no solution } $\angle$ is obtuse
  - one solution
  - two solutions } $\angle$ is acute

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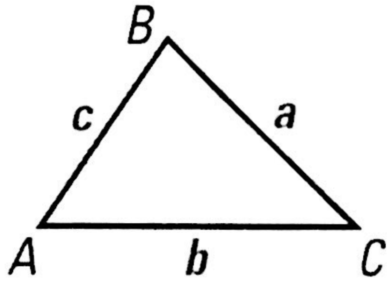
Notes:

How do we solve triangles in other cases?

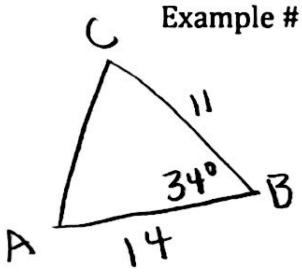
- Law of Cosines:
  - $a^2 = b^2 + c^2 - 2bc \cos A$
  - $b^2 = a^2 + c^2 - 2ac \cos B$
  - $c^2 = a^2 + b^2 - 2ab \cos C$



This can be used to solve triangles in the following cases:

- SAS: 
- SSS: 

Example #1: Solve ΔABC with $a = 11, c = 14$, and $B = 34°$



$b^2 = a^2 + c^2 - 2ac \cos B$

$b^2 = (11)^2 + (14)^2 - 2(11)(14) \cos 34°$

$b^2 \approx 61.7$

$\boxed{b \approx 7.9}$

$\frac{7.9}{\sin 34°} = \frac{11}{\sin A}$

$\boxed{\angle A \approx 51.1°}$

$\angle C = 180° - 34° - 51.1°$

$\boxed{\angle C \approx 94.9°}$

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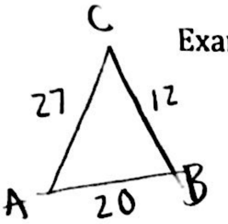


Example #2: Solve ΔABC with $a = 12, b = 27$, and $c = 20$

* Find largest angle first! (other two <u>must</u> be acute)

$27^2 = 20^2 + 12^2 - 2(20)(12)\cos B$

$729 = 544 - 480\cos B$

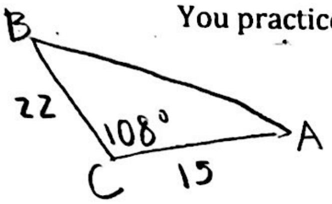
$-0.385 = \cos B$

$\boxed{\angle B \approx 112.7^\circ}$

$\frac{27}{\sin 112.7^\circ} = \frac{12}{\sin A}$ $\boxed{\angle A \approx 24.2^\circ}$

$\angle C = 180^\circ - 112.7^\circ - 24.2^\circ$

$\boxed{\angle C \approx 43.1^\circ}$



You practice: Solve ΔABC with $a = 22, b = 15$, and $C = 108°$

$c^2 = (22)^2 + (15)^2 - 2(22)(15)\cos 108^\circ$

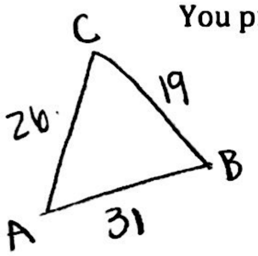
$c^2 = 912.95$

$\boxed{c \approx 30.2}$

$\frac{30.2}{\sin 108^\circ} = \frac{22}{\sin A}$ $\boxed{\angle A \approx 43.9^\circ}$

$\angle B = 180^\circ - 108^\circ - 43.9^\circ$

$\boxed{\angle B \approx 28.1^\circ}$



You practice: Solve ΔABC with $a = 19, b = 26$, and $c = 31$

$31^2 = 26^2 + 19^2 - 2(26)(19)\cos C$

$-76 = -988\cos C$

$\boxed{\angle C \approx 85.6^\circ}$

$\frac{31}{\sin 85.6^\circ} = \frac{19}{\sin A}$ $\boxed{\angle A \approx 37.7^\circ}$

$\angle B = 180^\circ - 85.6 - 37.7^\circ$

$\boxed{\angle B \approx 56.7^\circ}$

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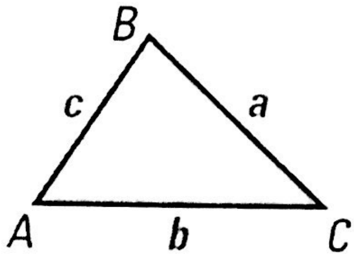
Notes:

Heron's Area Formula:

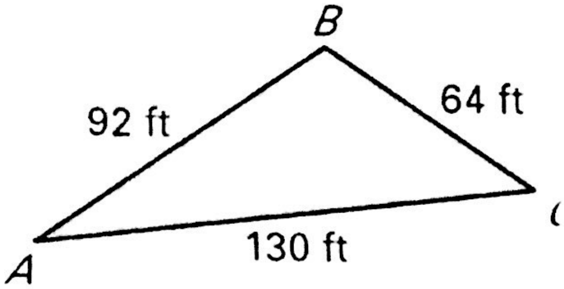
When only given the side lengths of a triangle, we can still find the area of ΔABC.

- Area = $\sqrt{s(s-a)(s-b)(s-c)}$
  - where $s = \frac{1}{2}(a+b+c)$

    ↳ semiperimeter (half)



Example #3: A triangular path around an exhibit at the zoo is shown. Find the area of the exhibit.



$s = \frac{1}{2}(92+64+130)$

$s = 143$

$A = \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{143(143-64)(143-130)(143-92)}$

$= \sqrt{143(79)(13)(51)}$

$= \sqrt{7489911}$

$A \approx \boxed{2{,}736.8 \text{ ft}^2}$