

NOTES: Section 11.6 Adding and Subtracting with Unlike Denominators

Goals: #1 - I can add and subtract rational expressions with unlike denominators.

Homework: LCD Matching Worksheet & Section 11.6 Worksheet



Warm Up:

1. Find the sum of the rational expression and simplify.

a. $\frac{4x}{7x+2} + \frac{5x}{7x+2}$

$$\frac{4x+5x}{7x+2}$$

$$\boxed{\frac{9x}{7x+2}} \quad \frac{3 \cdot 3 \cdot x}{(7x+2)}$$

$$\begin{array}{r} -10 \\ \wedge \\ -5 \quad 2 \\ x^2 - 5x + 2x - 10 \\ \hline x(x-5) + 2(x-5) \end{array}$$

b. $\frac{-10}{x^2+x-30} + \frac{x^2-3x}{x^2+x-30}$

$$\frac{x^2-3x-10}{x^2+x-30}$$

$$\frac{(x-5)(x+2)}{(x+6)(x-5)}$$

$$\boxed{\frac{x+2}{x+6}}$$

$$\begin{array}{r} -30 \\ \wedge \\ 6 \quad -5 \\ x^2 + 6x - 5x - 30 \\ \hline x(x+6) - 5(x+6) \end{array}$$

2. Find the difference of the rational expression and simplify.

a. $\frac{y^2}{y^2-16} - \frac{y+12}{y^2-16}$

$$\frac{y^2 - (y+12)}{y^2-16}$$

$$\frac{y^2 - y - 12}{y^2-16}$$

$$\frac{(y-4)(y+3)}{(y+4)(y-4)}$$

$$\boxed{\frac{y+3}{y+4}}$$

$$\begin{array}{r} -12 \\ \wedge \\ -4 \quad 3 \\ y^2 - 4y + 3y - 12 \\ \hline y(y-4) + 3(y-4) \end{array}$$

$$\begin{array}{r} 6 \\ \wedge \\ -3 \quad -2 \\ m^2 - 3m - 2m + 6 \\ \hline m(m-3) - 2(m-3) \end{array}$$

b. $\frac{2m^2}{m^2-5m+6} - \frac{4m}{m^2-5m+6}$

$$\frac{2m^2 - 4m}{m^2 - 5m + 6}$$

$$\frac{2m(m-2)}{(m-3)(m-2)}$$

$$\boxed{\frac{2m}{m-3}}$$

Exploration #1: Work with a partner and add the following fractions.

1. $\frac{3}{5} + \frac{1}{10}$

$$\frac{3 \cdot 2}{5 \cdot 2} = \frac{6}{10}$$

$$\frac{6}{10} + \frac{1}{10} = \boxed{\frac{7}{10}}$$

$$\begin{array}{r} \times 2 \quad \frac{7}{10} + \frac{1}{3} \quad \times 10 \\ \frac{7}{10} = \frac{21}{30} \quad \frac{1}{3} = \frac{10}{30} \\ \hline \frac{21}{30} + \frac{10}{30} = \boxed{\frac{31}{30}} \end{array}$$

3. $\frac{1}{2} + \frac{1}{8}$

$$\frac{1}{2} = \frac{4}{8}$$

$$\frac{4}{8} + \frac{1}{8} = \boxed{\frac{5}{8}}$$

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Notes:

As with fractions, to add or subtract

rational expressions with unlike denominators, we need to first rewrite the

expressions to have common denominators.

Then we can add or subtract the rational expressions.

Example #1: Find the least common denominator.

1. $\frac{x}{x-5}, \frac{2x^3}{x+7}$

① Factor: $(x-5)$
 $(x+7)$

② LCM: $(x-5)(x+7)$

2. $\frac{1}{36x}, \frac{3x+1}{9x^5}$

① Factor: $2 \cdot 2 \cdot 3 \cdot 3 \cdot x$
 $3 \cdot 3 \cdot x \cdot x \cdot x \cdot x \cdot x$

② LCM: $3 \cdot 3 \cdot x \cdot x \cdot x \cdot x \cdot x \cdot 2 \cdot 2$
 $36x^5$

You practice: Find the least common denominator.

1. $\frac{5x+9}{16x^3}, \frac{7}{24x^2}$

① Factor: $2 \cdot 2 \cdot 2 \cdot 2 \cdot x \cdot x \cdot x$
 $3 \cdot 2 \cdot 2 \cdot 2 \cdot x \cdot x$

② LCM: $2 \cdot 2 \cdot 2 \cdot 2 \cdot x \cdot x \cdot x \cdot 3$
 $48x^3$

2. $\frac{12}{x+1}, \frac{x}{x-1}$

① Factor: $(x+1)$
 $(x-1)$

② LCM: $(x+1)(x-1)$

Example #2: Find the missing numerator.

$$1. \frac{2}{3y} = \frac{?}{15y}$$

x 5

x 5

10

$$2. \frac{3x-7}{4x^2} = \frac{?}{36x^5}$$

x 9x³

x 9x³

9x³(3x-7)

27x⁴ - 63x³

$$3. \frac{y-1}{y} = \frac{?}{13y^2}$$

x 13y

x 13y

13y(y-1)

13y² - 13y

Notes:

We are ready to add or subtract rational expressions!

Step 1: Find LCD

Step 2: Rewrite each expression

Step 3: Add or subtract

Step 4: Factor & simplify

Example #3: Find the sum of the rational expression and simplify.

$$1. \frac{3}{15x^2} + \frac{1}{9x^3}$$

① LCD: ① Factor: 5 · 3 · x · x
3 · 3 · x · x · x

② LCM: 3 · 3 · x · x · x · 5

45x³

② Rewrite:

$$\frac{3}{15x^2} = \frac{9x}{45x^3}$$

x 3x

$$\frac{1}{9x^3} = \frac{5}{45x^3}$$

x 5

③ Add: $\frac{9x}{45x^3} + \frac{5}{45x^3} = \frac{9x+5}{45x^3}$

④ Simplify: $\frac{(9x+5)}{5 \cdot 3 \cdot 3 \cdot x \cdot x \cdot x}$

$$2. \frac{x+2}{x-1} + \frac{12}{x+6}$$

① LCD: ① Factor: (x-1)(x+6)

② LCM: (x-1)(x+6)

② Rewrite:

$$\frac{x+2}{x-1} = \frac{x^2+6x+2x+12}{(x-1)(x+6)}$$

x(x+6)

$$\frac{12}{x+6} = \frac{12x-12}{(x-1)(x+6)}$$

x(x-1)

③ Add:

$$\frac{x^2+8x+12}{(x-1)(x+6)} + \frac{12x-12}{(x-1)(x+6)} = \frac{x^2+20x}{(x-1)(x+6)}$$

④ Simplify: $\frac{x(x+20)}{(x-1)(x+6)}$

You practice: Find the sum of the rational expression and simplify.

1. $\frac{3}{12x^3} + \frac{x+1}{4x^3}$

① LCD: ① Factor: $3 \cdot 2 \cdot 2 \cdot x \cdot x \cdot x$
 $2 \cdot 2 \cdot x \cdot x \cdot x$

② LCM: $3 \cdot 2 \cdot 2 \cdot x \cdot x \cdot x$
 $12x^3$

③ Rewrite: $\frac{x+1}{4x^3} = \frac{3x+3}{12x^3}$

③ Add: $\frac{3}{12x^3} + \frac{3x+3}{12x^3} = \frac{3x+6}{12x^3}$

④ Simplify: $\frac{\cancel{3} \cdot (x+2)}{\cancel{3} \cdot 2 \cdot 2 \cdot x \cdot x \cdot x} = \frac{x+2}{4x^3}$

2. $\frac{x-3}{x+4} + \frac{6}{x-1}$

① LCD: Factor: $(x+4)(x-1)$
 LCM: $(x+4)(x-1)$

② Rewrite: $\frac{x-3}{x+4} = \frac{x^2-4x+3}{(x+4)(x-1)}$ $\frac{6}{x-1} = \frac{6x+24}{(x+4)(x-1)}$

③ Add: $\frac{x^2-4x+3}{(x+4)(x-1)} + \frac{6x+24}{(x+4)(x-1)} = \frac{x^2+2x+27}{(x+4)(x-1)}$

④ Simplify: $\frac{27}{x} = 2$

Example #4: Find the difference of the rational expression and simplify.

1. $\frac{7}{6x} - \frac{x+1}{8x^2}$

① LCD: ① $3 \cdot 2 \cdot x$
 $2 \cdot 2 \cdot 2 \cdot x \cdot x$
 ② $2 \cdot 2 \cdot 2 \cdot x \cdot x \cdot 3$
 $24x^2$

② Rewrite: $\frac{7}{6x} = \frac{28x}{24x^2}$ $\frac{x+1}{8x^2} = \frac{3x+3}{24x^2}$

③ Subtract: $\frac{28x}{24x^2} - \frac{3x+3}{24x^2} = \frac{25x-3}{24x^2}$

④ Simplify: $\frac{(25x-3)}{3 \cdot 2 \cdot 2 \cdot 2 \cdot x \cdot x}$

2. $\frac{x-5}{x+5} - \frac{x+2}{x-2}$

① LCD: ① $(x+5)(x-2)$
 ② $(x+5)(x-2)$

② Rewrite: $\frac{x-5}{x+5} = \frac{x^2-7x+10}{(x+5)(x-2)}$ $\frac{x+2}{x-2} = \frac{x^2+7x+10}{(x+5)(x-2)}$

③ Subtract: $\frac{x^2-7x+10}{(x+5)(x-2)} - \frac{x^2+7x+10}{(x+5)(x-2)} = \frac{-14x}{(x+5)(x-2)}$

④ Simplify: $\frac{-7 \cdot 2 \cdot x}{(x+5)(x-2)}$

You practice: Find the difference of the rational expression and simplify.

1. $\frac{3+4x}{4x^3} - \frac{1}{10x^2}$

① LCD: ① $2 \cdot 2 \cdot x \cdot x \cdot x$
 $5 \cdot 2 \cdot x \cdot x$
 ② $2 \cdot 2 \cdot x \cdot x \cdot x \cdot 5$
 $20x^3$

② Rewrite:
 $\frac{3+4x}{4x^3} \xrightarrow{\times 5} \frac{15+20x}{20x^3}$
 $\xrightarrow{\times 5}$

$\frac{1}{10x^2} \xrightarrow{\times 2x} \frac{2x}{20x^3}$
 $\xrightarrow{\times 2x}$

③ Subtract:

$\frac{15+20x}{20x^3} - \frac{2x}{20x^3} = \boxed{\frac{15+18x}{20x^3}}$

④ Simplify:

$\frac{3(5+6x)}{5 \cdot 2 \cdot 2 \cdot x \cdot x \cdot x}$

2. $\frac{2x}{x-1} - \frac{7x}{x+4}$

① LCD: ① $(x-1)$
 $(x+4)$
 ② $(x-1)(x+4)$

② Rewrite:
 $\frac{2x}{x-1} \xrightarrow{\times (x+4)} \frac{2x^2+8x}{(x-1)(x+4)}$
 $\xrightarrow{\times (x+4)}$
 $\frac{7x}{x+4} \xrightarrow{\times (x-1)} \frac{7x^2-7x}{(x-1)(x+4)}$
 $\xrightarrow{\times (x-1)}$

③ Subtract:

$\frac{2x^2+8x}{(x-1)(x+4)} - \frac{7x^2-7x}{(x-1)(x+4)} =$

$\boxed{\frac{-5x^2+15x}{(x-1)(x+4)}}$

④ Simplify:

$\frac{-5x(x-3)}{(x-1)(x+4)}$