Chapter 10 Review Worksheet

Name: KEY

For the given configuration, determine how many different license plates are possible if (a) digits and letters can be repeated, and (b) digits and letters cannot be repeated.

1.) 1 letter followed by 6 digits

a.) $\frac{26}{L} \cdot \frac{10}{\#} \cdot \frac{10}{\#} \cdot \frac{10}{\#} \cdot \frac{10}{\#} \cdot \frac{10}{\#} \cdot \frac{10}{\#}$

$= 26,000,000$ plates

b.) $\frac{26}{L} \cdot \frac{10}{\#} \cdot \frac{9}{\#} \cdot \frac{8}{\#} \cdot \frac{7}{\#} \cdot \frac{6}{\#} \cdot \frac{5}{\#}$

$= 3,931,200$ plates

2.) 2 digits followed by 5 letters

a.) $\frac{10}{\#} \cdot \frac{10}{\#} \cdot \frac{26}{L} \cdot \frac{26}{L} \cdot \frac{26}{L} \cdot \frac{26}{L} \cdot \frac{26}{L}$

$= 1,118,137,600$ plates

b.) $\frac{10}{\#} \cdot \frac{9}{\#} \cdot \frac{26}{L} \cdot \frac{25}{L} \cdot \frac{24}{L} \cdot \frac{23}{L} \cdot \frac{22}{L}$

$= 710,424,000$ plates

Find the number of <u>distinguishable</u> permutations of the letters in the word.

3.) SOUTH $5!$

$= 120$ permutations

4.) DELAWARE $\frac{8!}{2!2!}$

$= 10,080$ permutations

5.) MISSISSIPPI $\frac{11!}{4!4!2!}$

$= 34,620$ permutations

6.) A men's department store sells 3 different suit jackets, 6 different shirts, 8 different ties, and 4 different pairs of pants. How many different suits consisting of a jacket, shirt, tie, and pants are possible?

$\frac{3}{J} \cdot \frac{6}{S} \cdot \frac{8}{T} \cdot \frac{4}{P} =$ 576 suit combinations

7.) You are placing 12 pictures on separate pages in an album.

a.) How many different ways can you order the 12 pictures in the album?

$12 \cdot 11 \cdot 10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1$ OR $12!$

$=$ 479,001,600 ways

b.) How many different ways can 4 of the 12 pictures be placed on the first four pages?

${}_{12}P_4$ OR $12 \cdot 11 \cdot 10 \cdot 9$

$=$ 11,880 ways

Find the number of possible 5-card hands that contain the cards specified. The cards are taken from a standard 52-card deck.

8.) 3 face cards and 2 tens

${}_{12}C_3 \cdot {}_4C_2$

$=$ 1320 hands

9.) 2 spades and 3 red cards

${}_{13}C_2 \cdot {}_{26}C_3$

$=$ 202,800 hands

10.) at most 2 clubs

0 CL, 5 OC
1 CL, 4 OC
2 CL, 3 OC

${}_{13}C_0 \cdot {}_{39}C_5 + {}_{13}C_1 \cdot {}_{39}C_4 + {}_{13}C_2 \cdot {}_{39}C_3$

$575,757 + 1,069,263 + 712,842$

$=$ 2,357,862 hands

11.) at least 1 diamond

1D, 4 OC
2D, 3 OC
3D, 2 OC
4D, 1 OC
5D, 0 OC

NOT: 0D, 5 OC

Total − (0D, 5 OC)

${}_{52}C_5 - ({}_{13}C_0 \cdot {}_{39}C_5)$

$2,598,960 - 575,757 =$ 2,023,203 hands

Use the binomial theorem to write the binomial expansion.

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1
1 6 15 20 15 6 1

12.) $(4x-1)^6$

$1(4x)^6(-1)^0 + 6(4x)^5(-1)^1 + 15(4x)^4(-1)^2 + 20(4x)^3(-1)^3 + 15(4x)^2(-1)^4 + 6(4x)^1(-1)^5 + 1(4x)^0(-1)^6$

$4096 \cdot x^6 + 6 \cdot 1025 \cdot -1 \cdot x^5 + 15 \cdot 256 \cdot 1 \cdot x^4 + 20 \cdot 64 \cdot -1 \cdot x^3 + 15 \cdot 16 \cdot 1 \cdot x^2 + 6 \cdot 4 \cdot -1 \cdot x + 1 \cdot 1$

$$\boxed{4096x^6 - 6144x^5 + 3840x^4 - 1280x^3 + 240x^2 - 24x + 1}$$

13.) $(8x+y)^4$

$1(8x)^4(y)^0 + 4(8x)^3(y)^1 + 6(8x)^2(y)^2 + 4(8x)^1(y)^3 + 1(8x)^0(y)^4$

$4096 \cdot x^4 + 4 \cdot 512 \cdot x^3 \cdot y + 6 \cdot 64 \cdot x^2 \cdot y^2 + 4 \cdot 8 \cdot x \cdot y^3 + y^4$

$$\boxed{4096x^4 + 2048x^3y + 384x^2y^2 + 32xy^3 + y^4}$$

14.) The student council consists of 6 seniors, 5 juniors, 4 sophomores, and 3 freshman.

18 total students

a.) How many different committees of exactly 2 seniors and 2 juniors can be chosen?

(order does NOT matter)

$${}_6C_2 \cdot {}_5C_2 = \boxed{150 \text{ committees}}$$

b.) How many different committees of *at most* 4 students can be chosen?

1 S
2 S
3 S
4 S

4 students: 3 students: 2 students: 1 student:

$${}_{18}C_4 + {}_{18}C_3 + {}_{18}C_2 + {}_{18}C_1 = \boxed{4047 \text{ committees}}$$

15.) An ice cream vendor sells 15 flavors of ice cream. You want to sample *at least* 4 of the flavors. How many different combinations of ice cream flavors can you sample?

4 FL
5 FL
6 FL
⋮
14 FL

Total − (0 FL, 1 FL, 2 FL, 3 FL)

① Try OR ② not try →

$$2^{15} - ({}_{15}C_0 + {}_{15}C_1 + {}_{15}C_2 + {}_{15}C_3)$$

$$32{,}768 - 576 = \boxed{32{,}192 \text{ combinations}}$$

16.) A teacher is holding tryouts for the school play. There are 15 students trying out for 7 parts in the play. Each student can play each part. In how many ways can the teacher select the students?

(order does NOT matter)

$${}_{15}C_7 = \boxed{6{,}435 \text{ ways}}$$

You have an equally likely chance of choosing any integer from 1 through 30. Find the probability of the given event.

17.) An even number is chosen

$\frac{15}{30} = \boxed{\frac{1}{2}}$

18.) A multiple of 5 is chosen

$\frac{6}{30} = \boxed{\frac{1}{5}}$

19.) A factor of 60 is chosen

$\boxed{\frac{11}{30}}$ (1, 2, 3, 4, 5, 6, 10, 12, 15, 20, 30)

20.) A prime number is chosen

$\frac{10}{30} = \boxed{\frac{1}{3}}$ (2, 3, 5, 7, 11, 13, 17, 19, 23, 29)

You randomly select a marble from a bag. The bag contains 7 black, 14 red, 6 white, and 13 blue marbles. Find the indicated odds.

40 total marbles

21.) In favor of choosing black

$\boxed{7:33}$

22.) In favor of choosing red or white

20:20

$\boxed{1:1}$

22.) Against choosing white

34:6

$\boxed{17:3}$

23.) Against choosing blue or white

$\boxed{21:19}$

24.) Out of 250 work days, a commuter arrived at work on time 47 times on Mondays, 43 times on Tuesdays, 48 times on Wednesdays, 39 times on Thursdays, and 40 times on Fridays. For a randomly selected work day, what is the probability that the commuter arrived at work on time? The commuter arrived on time 217 out of the 250 total days

$P(\text{arrived on time}) = \boxed{\frac{217}{250}} \approx \boxed{0.868}$

Find the indicated probability.

25.) $P(A) = 0.3$
$P(B) = 0.6$
$P(A \text{ or } B) = ?$
$P(A \text{ and } B) = 0.1$

$P(A \text{ or } B) = 0.3 + 0.6 - 0.1$
$P(A \text{ or } B) = \boxed{0.8}$

26.) $P(A) = 35\%$
$P(B) = ?$
$P(A \text{ or } B) = 80\%$
$P(A \text{ and } B) = 20\%$

$80\% = 35\% + P(B) - 20\%$
$P(B) = \boxed{65\%}$

27.) $P(A) = ?$
$P(\bar{A}) = \frac{2}{5}$

$P(A) = 1 - \frac{2}{5}$
$P(A) = \boxed{\frac{3}{5}}$

28.) A and B are independent.
$P(A) = 0.15$
$P(B) = 0.6$
$P(A \text{ and } B) = ?$

$P(A \text{ and } B) = 0.15 \cdot 0.6$
$P(A \text{ and } B) = \boxed{0.09}$

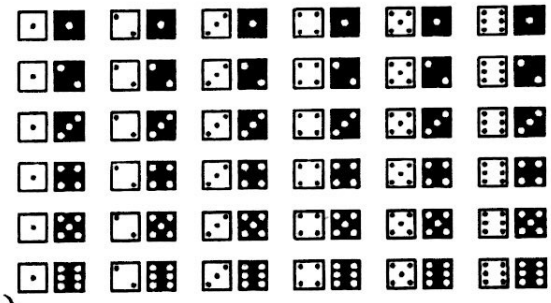
29.) A and B are dependent.
$P(A) = 60\%$
$P(B|A) = ?$
$P(A \text{ and } B) = 25\%$

$25\% = 60\% \cdot P(B|A)$
$P(B|A) = \boxed{41.67\%}$

30.) A and B are dependent.
$P(A) = ?$
$P(B|A) = 0.4$
$P(A \text{ and } B) = 0.36$

$0.36 = P(A) \cdot 0.4$
$P(A) = \boxed{0.9}$

Two six-sided dice are rolled. Find the probability of the given event.



31.) The sum is greater than 4.

$\frac{30}{36} = \boxed{\frac{5}{6}}$

32.) The sum is 6 or 11.

$P(S6 \text{ or } S11) = \frac{5}{36} + \frac{2}{36} = \boxed{\frac{7}{36}}$
$P(S6) + P(S11)$

(or count) $\boxed{\frac{7}{36}}$

33.) The sum is neither 5 nor 9.

$P(\bar{S5} \text{ or } \bar{S9}) = 1 - \frac{4}{36} - \frac{4}{36} = \boxed{\frac{7}{9}}$
$= 1 - P(S5) - P(S9)$

(or (count)) $\frac{28}{36} = \boxed{\frac{7}{9}}$

34.) The sum is greater than 7 and less than 11.

(count) $\frac{12}{36} = \boxed{\frac{1}{3}}$

Find the probability of randomly selecting the given marbles from a bag of 5 red, 8 green, and 3 blue marbles if (a) you replace the first marble before drawing the second and (b) you do *not* replace the first marble.

35.) red, then green

a) $\frac{5}{16} \cdot \frac{8}{16} = \boxed{\frac{5}{32}}$

b) $\frac{5}{16} \cdot \frac{8}{15} = \boxed{\frac{1}{6}}$

36.) blue, then red.

a) $\frac{3}{16} \cdot \frac{5}{16} = \boxed{\frac{15}{256}}$

b) $\frac{3}{16} \cdot \frac{5}{15} = \boxed{\frac{1}{16}}$

37.) green then green.

a) $\frac{8}{16} \cdot \frac{8}{16} = \boxed{\frac{1}{4}}$

b) $\frac{8}{16} \cdot \frac{7}{15} = \boxed{\frac{7}{30}}$

Let *n* be a randomly selected integer from 1 to 20. Find the indicated probability.

38.) *n* is 2 given that it is even

10 even #s

$\boxed{\frac{1}{10}}$

39.) *n* is odd given that it is prime

2, 3, 5, 7, 11, 13, 17, 19 8 prime #s

$\boxed{\frac{7}{8}}$

40.) *n* is 5 given that it is less than 8

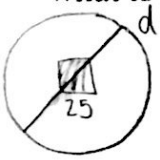
1, 2, 3, 4, 5, 6, 7 7 #s < 8

$\boxed{\frac{1}{7}}$

41.) You and 4 friends are in line at lunch and are each selecting a beverage. There are 5 types of beverages available. What is the probability that all of you will select different beverages?

$\frac{1}{5} \cdot \frac{1}{5} \cdot \frac{1}{5} \cdot \frac{1}{5} \cdot \frac{1}{5} = \boxed{\frac{1}{3125}} \approx \boxed{0.00032}$

42.) A parachuter is attempting to land within a square in the middle of a circular landing area. The square has sides 25 feet long, and the diameter of the landing area is 40 feet. If the parachuter is equally likely to first touch the ground at any point within the landing area, what is the probability that the parachuter first touches ground within the square?



■ Area = $625\ ft^2$

○ Area = $\pi(20)^2\ ft^2$
$= 400\pi\ ft^2$

$\frac{625}{400\pi} \approx \boxed{0.497}$

43.) A high school has an enrollment of 1800 students. There are 1050 females enrolled in the school. The high school has 1200 students who are involved in an after-school activity, 725 of whom are female. What is the probability that a randomly selected student at the school is a female who is not involved in an after school activity?

$1050 - 725 = 325$ females NOT involved in after school activity

$$\frac{325}{1800} = \boxed{\frac{13}{72}} \approx \boxed{0.1806}$$

44.) A pet store has 18 light green parakeets (5 females and 13 males) and 25 sky blue parakeets (15 females and 10 males). You randomly choose one of the parakeets. What is the probability that it is a male or a sky blue parakeet? 43 total parakeets

$$P(M \text{ or } P) = \frac{23}{43} + \frac{25}{43} - \frac{10}{43} = \boxed{\frac{38}{43}}$$

$P(M) + P(P) - P(\text{BOTH})$

45.) A tennis player wins a match 55% of the time when she serves first and 47% of the time when her opponent serves first. The player who serves first is determined by a coin toss before the match. What is the probability that the player wins a given match?

Event 1: win coin toss · wins:

$\frac{1}{2} \cdot 0.55$

+

Event: lose coin toss · wins:

$\frac{1}{2} + 0.47$

$= 0.275 + 0.235$

$= \boxed{0.51}$ or $\boxed{51\%}$