Name: KEY Hour: _____ Date: _____

NOTES: Section 7.5 – Apply Properties of Logarithms

Goals: #1 - I can use properties of a logarithm to evaluate logarithms.

#2 - I can use properties of a logarithm to expand and condense logarithms.

#3 - I can use the change of base formula to evaluate logarithms.

Homework: Lesson 7.5 Worksheet

Warm Up:

1. Rewrite the equation in its alternate form.
   a. $\log 10{,}000 = 4$

   $10^4 = 10{,}000$

   b. $e^7 = x - 3$

   $\ln(x-3) = 7$

2. Evaluate the logarithm without a calculator.
   a. $\log_6 216$

   $6^? = 216$

   3

   b. $\log_{16} \frac{1}{4}$

   $16^? = \frac{1}{4}$

   $-\frac{1}{2}$

   c. $\log_{1/4} 16$

   $\frac{1}{4}^? = 16$

   -2

3. Find the inverse of the function.
   a. $y = \log(x - 2)$

   $10^y = x - 2$

   $10^x = y - 2$

   $y = 10^x + 2$

   b. $y = (0.4)^x$

   $\log_{0.4} y = x$

   $y = \log_{0.4} x$

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Exploration #1: Work with a partner and answer the following questions.

1. Let $x = \log_b m$ and $y = \log_b n$

   a. Rewrite these logarithmic equations in exponential form:

   $x = \log_b m \rightarrow$ $b^x = m$ $\qquad$ $y = \log_b n \rightarrow$ $b^y = n$

   b. Now multiply mn:

   $mn =$ $b^x \cdot b^y = b^{x+y}$

   c. Rewrite this exponential equation in logarithmic form:

   $mn = b^{x+y} \rightarrow$ $\log_b(mn) = x + y$
   $= \log_b m + \log_b n$

2. Let $x = \log_b m$ and $y = \log_b n$

   a. Rewrite these logarithmic equations in exponential form:

   $x = \log_b m \rightarrow$ $b^x = m$ $\qquad$ $y = \log_b n \rightarrow$ $b^y = n$

   b. Now divide $\frac{m}{n}$:

   $\frac{m}{n} = \frac{b^x}{b^y} = b^{x-y}$

   c. Rewrite this exponential equation in logarithmic form:

   $\frac{m}{n} = b^{x-y} \rightarrow$ $\log_b \frac{m}{n} = x - y$
   $= \log_b m - \log_b n$

Notes:

Properties of Logarithms:

Product Property	$\log_b(mn) = \log_b m + \log_b n$
Quotient Property	$\log_b\left(\frac{m}{n}\right) = \log_b m - \log_b n$
Power Property	$\log_b(m^n) = n \cdot \log_b m$

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Example #1: Use $\log_4 3 \approx 0.792$ and $\log_4 7 \approx 1.404$ to evaluate the logarithm.

1. $\log_4 \frac{3}{7}$

$\log_4 3 - \log_4 7$

$0.792 - 1.404$

$\approx \boxed{-0.612}$

2. $\log_4 21$

$\log_4 (7 \cdot 3)$

$\log_4 7 + \log_4 3$

$1.404 + 0.792$

$\approx \boxed{2.196}$

3. $\log_4 49$

$\log_4 7^2$

$2 \cdot \log_4 7$

$2(1.404)$

$\approx \boxed{2.808}$

Example #2: Use the properties of logarithms to expand or condense the following expressions.

1. Expand $\log_6 \frac{5x^3}{y}$

$\log_6 5x^3 - \log_6 y$

$\log_6 5 + \log_6 x^3 - \log_6 y$

$\boxed{\log_6 5 + 3\log_6 x - \log_6 y}$

2. Condense $\log 2 + 3\log 3 - \log 9$

$\log 2 + \log 3^3 - \log 9$

$\log 2 + \log 27 - \log 9$

$\log (2 \cdot 27) - \log 9$

$\log (54) - \log 9$

$\log \left(\frac{54}{9}\right)$

$\boxed{\log 6}$

You practice: Use the properties of logarithms to expand or condense the following expressions.

1. Expand $\log_3 \frac{7x^2}{y}$

$\log_3 7x^2 - \log_3 y$

$\log_3 7 + \log_7 x^2 - \log_3 y$

$\boxed{\log_3 7 + 2\log_7 x - \log_3 y}$

2. Condense $\ln 4 + 3\ln 3 - \ln 12$

$\ln 4 + \ln 3^3 - \ln 12$

$\ln 4 + \ln 27 - \ln 12$

$\ln (4 \cdot 27) - \ln 12$

$\ln 108 - \ln 12$

$\ln \left(\frac{108}{12}\right)$

$\boxed{\ln 9}$

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Notes:

Change-of-Base Formula:

Since our calculators can only evaluate logarithms with base 10, we can use the change-of-base formula to evaluate ANY logarithm using a calculator.

$$\log_b C = \frac{\log C}{\log b}$$

Example #3: Use the change-of-base formula to evaluate the logarithm.

1. $\log_3 8$

$$\frac{\log 8}{\log 3} \approx \boxed{1.8928}$$

2. $\log_6 11$

$$\frac{\log 11}{\log 6} \approx \boxed{1.3383}$$

Extra practice:

1. Expand $\log_7 \frac{3x^2}{5y^3}$

$$\log_7 3x^2 - \log_7 5y^3$$
$$\log_7 3 + \log_7 x^2 - \log_7 5 - \log_7 y^3$$
$$\boxed{\log_7 3 + 2\log_7 x - \log_7 5 - 3\log_7 y}$$

2. Condense $5\log_4 2 + 7\log_4 x - 4\log_4 y$

$$\log_4 2^5 + \log_4 x^5 - \log_4 y^4$$
$$\log_4 32 + \log_4 x^5 - \log_4 y^4$$
$$\log_4 32x^5 - \log_4 y^4$$
$$\boxed{\log_4 \frac{32x^5}{y^4}}$$