

NOTES: Section 4.5 – Solve Quadratic Equations by Finding Square Roots

Goals: #1 - I can write square root expressions in simplest radical form by simplifying and rationalizing the denominator. 😎 😐 😞

#2 - I can solve quadratics in the form $ax^2 + c$ by finding square roots.

Homework: Lesson 4.5 Worksheet

Warm Up:

1. Factor the following expressions:

a. $8r^2 + 6r - 5$ $8 \cdot -5 = -40$
 $10 \quad -4 = 6$
 $8r^2 - 4r + 10r - 5$
 $4r(2r-1) + 5(2r-1)$
 $(2r-1)(4r+5)$

b. $9m^2 + 30mn + 25n^2$
 $\downarrow \quad \downarrow \quad \downarrow$
 $(3m)^2 + 2(3m \cdot 5n) + (5n)^2$
 $(3m + 5n)^2$

2. Solve the following equation.

a. $5x^2 + x - 4 = 0$ $5 \cdot -4 = -20$
 $5 \quad -4 = 1$
 $5x^2 + 5x - 4x - 4 = 0$
 $5x(x+1) - 4(x+1) = 0$
 $(x+1)(5x-4) = 0$
 $x = -1$

$5x - 4 = 0$
 $5x = 4$
 $x = \frac{4}{5}$
 3. $\sqrt{\frac{7}{16}}$
 $= \frac{\sqrt{7}}{\sqrt{16}}$
 $\frac{\sqrt{7}}{4}$

Review: Simplify the expression.

1. $\sqrt{80}$
 $= \sqrt{16} \cdot \sqrt{5}$
 $4\sqrt{5}$

2. $\sqrt{6} \cdot \sqrt{21}$
 $= \sqrt{126}$
 $= \sqrt{9} \cdot \sqrt{14}$
 $3\sqrt{14}$

Notes:

Recall some properties of square roots.

- Product Property: $\sqrt{ab} = \sqrt{a} \cdot \sqrt{b}$
- Quotient Property: $\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$

Perfect Squares:	4	9	16	25	36	49	64	81	100	121	144
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Example #1: Simplify the expression.

$$\begin{aligned}
 1. \quad & 3\sqrt{20} \cdot \sqrt{40} \\
 & = 3\sqrt{800} \\
 & = 3 \cdot \sqrt{100} \cdot \sqrt{4} \cdot \sqrt{2} \\
 & = 3 \cdot 10 \cdot 2 \cdot \sqrt{2} \\
 & = \boxed{60\sqrt{2}}
 \end{aligned}$$

$$\begin{aligned}
 2. \quad & \sqrt{180} \\
 & = \sqrt{36} \cdot \sqrt{5} \\
 & = \boxed{6\sqrt{5}}
 \end{aligned}$$

$$\begin{aligned}
 3. \quad & \sqrt{\frac{11}{25}} \\
 & = \frac{\sqrt{11}}{\sqrt{25}} \\
 & = \boxed{\frac{\sqrt{11}}{5}}
 \end{aligned}$$

$$\begin{aligned}
 4. \quad & \sqrt{7} \cdot \sqrt{35} \\
 & = \sqrt{245} \\
 & = \sqrt{49} \cdot \sqrt{5} \\
 & = \boxed{7\sqrt{5}}
 \end{aligned}$$

Exploration #1: Work with a partner. Simplify the expression.

$$\begin{aligned}
 1. \quad & \sqrt{\frac{17}{12}} \\
 & = \frac{\sqrt{17}}{\sqrt{12}} \cdot \frac{\sqrt{12}}{\sqrt{12}} \\
 & = \frac{\sqrt{204}}{12} = \frac{\sqrt{4 \cdot 51}}{12} = \frac{2\sqrt{51}}{12} = \boxed{\frac{\sqrt{51}}{6}}
 \end{aligned}$$

$$\begin{aligned}
 2. \quad & \sqrt{\frac{6}{5}} \\
 & = \frac{\sqrt{6}}{\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}} \\
 & = \boxed{\frac{\sqrt{30}}{5}}
 \end{aligned}$$

Notes:

When we get a radical ($\sqrt{\quad}$) symbol in our denominator, we need to rationalize the denominator.

- $\sqrt{a} \rightarrow$ multiply by \sqrt{a}

Examples: $\frac{6}{\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}} = \frac{6\sqrt{5}}{(\sqrt{5})^2} = \boxed{\frac{6\sqrt{5}}{5}}$

radical in denominator? Nope!

- $a + \sqrt{b}$ or $a - \sqrt{b} \rightarrow$ multiply by conjugate

Examples: $\frac{4}{5-\sqrt{2}} \cdot \frac{5+\sqrt{2}}{5+\sqrt{2}} = \frac{4(5+\sqrt{2})}{25+5\sqrt{2}-5\sqrt{2}-(\sqrt{2})^2}$

$$\begin{aligned}
 & = \frac{20+4\sqrt{2}}{25-4} \\
 & = \boxed{\frac{20+4\sqrt{2}}{21}}
 \end{aligned}$$

radical in denominator? Nope!

Example #2: Simplify the expression.

$$1. \sqrt{\frac{5}{2}}$$

$$= \frac{\sqrt{5}}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}}$$

$$= \boxed{\frac{\sqrt{10}}{2}} \checkmark$$

radical in denominator?
Nope!

$$2. \frac{3}{7+\sqrt{2}} \cdot \frac{7-\sqrt{2}}{7-\sqrt{2}}$$

$$= \frac{21-3\sqrt{2}}{49-2}$$

$$= \boxed{\frac{21-3\sqrt{2}}{47}} \checkmark$$

You practice: Simplify the expression.

$$1. \sqrt{\frac{19}{21}}$$

$$= \frac{\sqrt{19}}{\sqrt{21}} \cdot \frac{\sqrt{21}}{\sqrt{21}}$$

$$= \boxed{\frac{\sqrt{399}}{21}} \checkmark$$

$$2. \sqrt{10} \cdot \sqrt{15}$$

$$= \sqrt{150}$$

$$= \sqrt{25} \cdot \sqrt{6}$$

$$= \boxed{5\sqrt{6}}$$

$$3. \frac{2}{4+\sqrt{11}} \cdot \frac{4-\sqrt{11}}{4-\sqrt{11}}$$

$$= \frac{2(4-\sqrt{11})}{16-11}$$

$$= \boxed{\frac{8-2\sqrt{11}}{5}}$$

Example #3: Solve the equation.

$$1. 3x^2 + 5 = 41$$

$$\quad -5 \quad -5$$

$$\frac{3x^2}{3} = \frac{36}{3}$$

$$x^2 = 12$$

$$\sqrt{x^2} = \sqrt{12}$$

$$x = \sqrt{4} \cdot \sqrt{3} = \boxed{\pm 2\sqrt{3}}$$

$$2. 2x^2 - 15 = 65$$

$$\quad +15 \quad +15$$

$$\frac{2x^2}{2} = \frac{80}{2}$$

$$x^2 = 40$$

$$\sqrt{x^2} = \sqrt{40}$$

$$x = \sqrt{4} \cdot \sqrt{10}$$

$$x = \boxed{\pm 2\sqrt{10}}$$

You practice: Solve the equation.

$$1. z^2 - 7 = 29$$

$$\quad +7 \quad +7$$

$$\sqrt{z^2} = \sqrt{36}$$

$$\boxed{z = \pm 6}$$

$$2. \frac{3(x-2)^2}{3} = \frac{40}{3}$$

$$(x-2)^2 = \frac{40}{3}$$

$$\sqrt{(x-2)^2} = \sqrt{\frac{40}{3}}$$

$$x-2 = \frac{\sqrt{40}}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}}$$

$$x-2 = \frac{\sqrt{120}}{3}$$

$$\quad +2 \quad +2$$

$$x = \frac{\sqrt{4} \cdot \sqrt{30}}{3} + 2$$

$$x = \boxed{\frac{\pm 2\sqrt{30}}{3} + 2}$$

Name: _____ Hour: _____ Date: _____

Example #4: When an object is dropped, its height h (in feet) above the ground after t seconds can be modeled by the function $h = -16t^2 + h_0$ where h_0 is the object's initial height (in feet).

For a science competition, students must design a container that prevents an egg from breaking when dropped from a height of 50 feet. How long does the container take to hit the ground?

$$h = -16t^2 + 50$$

$$0 = -16t^2 + 50$$

$$\begin{array}{r} -50 \\ -50 \end{array}$$

$$\frac{-50}{-16} = \frac{-16t^2}{-16}$$

$$3.125 = t^2$$

$$\sqrt{3.125} = \sqrt{t^2}$$

$$t \approx \boxed{1.77 \text{ seconds}}$$

