Name: KEY Hour: ____________ Date: ____________

NOTES: Section 13.5 – Apply the Law of Sines

Goals: #1 - I can solve a triangle using the Law of Sines (need to know at least one angle and the opposite side).

#2 - I can find the area of a triangle when given two sides and that included angle.

Homework: Lesson 13.5 Worksheet

Warm Up:

1. Evaluate the expression. Give your answer in both radians and degrees. NO CALCULATOR.

$-90° \le \theta \le 90°$ a. $\sin^{-1}\frac{\sqrt{2}}{2}$ $0° \le \theta \le 180°$ b. $\cos^{-1} -\frac{\sqrt{3}}{2}$ $-90° < \theta < 90°$ c. $\tan^{-1}\frac{\sqrt{3}}{3}$

$\boxed{\theta = 45°, \frac{\pi}{4}}$ $\boxed{\theta = 150°, \frac{5\pi}{6}}$ $\frac{y}{x}$ $\frac{\frac{1}{2}}{\frac{\sqrt{3}}{2}} = \frac{1}{2} \cdot \frac{2}{\sqrt{3}} = \frac{1}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{3}}{3}$

$\boxed{\theta = 30°, \frac{\pi}{6}}$

2. Solve the equation $\tan \theta = -2.5;\ 90° < \theta < 180°$

$\tan^{-1}(\tan \theta) = \tan^{-1}(-2.5)$

$\theta = -68.2°$

$\theta = 180° - 68.2°$

$\boxed{\theta = 111.8°}$

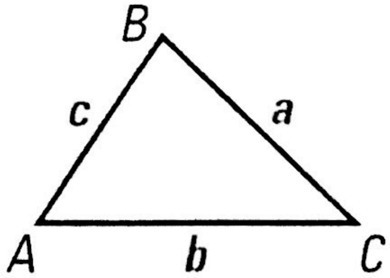
Notes:

How do we solve triangles with NO right angles?

- Law of Sines:

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$



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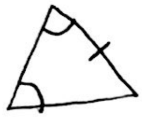
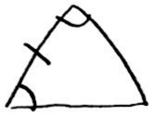
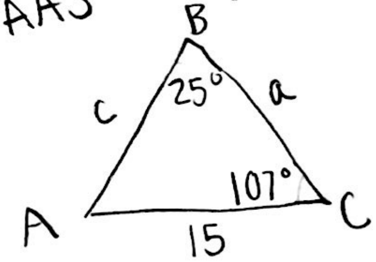
This can be used to solve triangles when two angles and the length of any side are known.

- AAS: 
- ASA: 
- SSA:  -could have NO SOLUTION
  -could have 2 solutions

AAS

Example #1: Solve ΔABC with $C = 107°, B = 25°$, and $b = 15$



$\angle A = 180° - 107° - 25°$

$\boxed{\angle A = 48°}$

$\frac{a}{\sin 48°} = \frac{15}{\sin 25°}$

$a \cdot \sin 25° = 15 \cdot \sin 48°$

$\boxed{a \approx 26.4}$

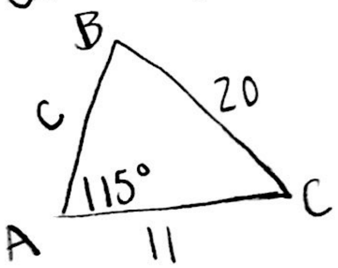
$\frac{c}{\sin 107°} = \frac{15}{\sin 25°}$

$c \cdot \sin 25° = 15 \cdot \sin 107°$

$\boxed{c = 33.9}$

SSA

Example #2: Solve ΔABC with $= 115°, a = 20$, and $b = 11$



$\frac{20}{\sin 115°} = \frac{11}{\sin B}$

$20 \cdot \sin B = 11 \cdot \sin 115°$

$\sin B = 0.498$

$\angle B = \sin^{-1}(0.498)$

$\boxed{\angle B \approx 29.9°}$

$\angle C = 180° - 115° - 29.9°$

$\boxed{\angle C = 35.1°}$

$\frac{20}{\sin 115°} = \frac{c}{\sin 35.1°}$

$c \cdot \sin 115° = 20 \cdot \sin 35.1°$

$\boxed{c \approx 12.7}$

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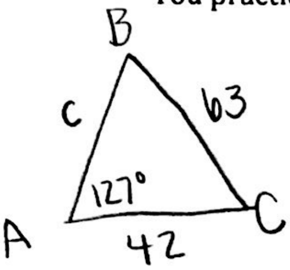
You practice: Solve ΔABC with $A = 127°$, $a = 63$, and $b = 42$



$\frac{63}{\sin 127°} = \frac{42}{\sin B}$

$63 \cdot \sin B = 42 \cdot \sin 127°$

$\sin B = 0.532$

$\angle B = \sin^{-1}(0.532)$

$\boxed{\angle B = 32.2°}$

$\angle C = 180° - 127° - 32.2°$

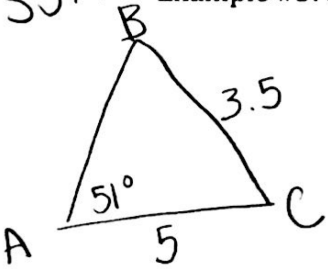
$\boxed{\angle C = 20.8°}$

$\frac{63}{\sin 127°} = \frac{c}{\sin 20.8°}$

$c \cdot \sin 127° = 63 \cdot \sin 20.8°$

$\boxed{c \approx 28}$

SSA **Example #3:** Solve ΔABC with $A = 51°$, $a = 3.5$, and $b = 5$



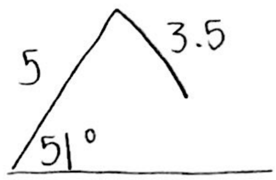
$\frac{3.5}{\sin 51°} = \frac{5}{\sin B}$

$3.5 \cdot \sin B = 5 \cdot \sin 51°$

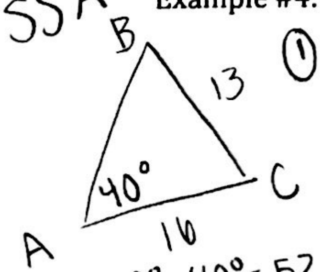
$\sin B = 1.11$

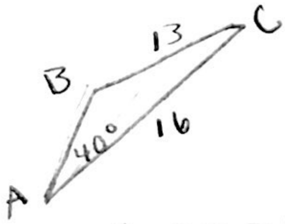
$\angle B = \sin^{-1}(1.11)$

$\boxed{\text{NO SOLUTION}}$



SSA **Example #4:** Solve ΔABC with $A = 40°$, $a = 13$, and $b = 16$





① $\frac{13}{\sin 40°} = \frac{16}{\sin B}$

$13 \cdot \sin B = 16 \cdot \sin 40°$

$\sin B = 0.791$

$\angle B = \sin^{-1}(0.791)$

$\boxed{\angle B = 52.3°}$

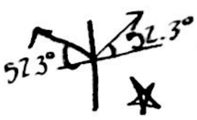
$\angle C = 180° - 40° - 52.3°$

$\boxed{\angle C = 87.7°}$

$\frac{13}{\sin 40°} = \frac{c}{\sin 87.7°}$

$c \cdot \sin 40° = 13 \cdot \sin 87.7°$

$\boxed{c \approx 20.2}$



② $\angle B = 180° - 52.3°$

$\boxed{\angle B = 127.7°}$

$\angle C = 180° - 40° - 127.7°$

$\boxed{\angle C = 12.3°}$

$\frac{13}{\sin 40°} = \frac{b}{\sin 12.3°}$

$b \cdot \sin 40° = 13 \cdot \sin 12.3°$

$\boxed{b \approx 4.3}$

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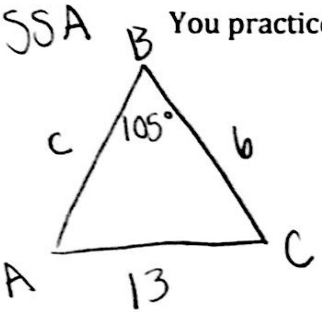
You practice: Solve ΔABC with $B = 105°$, $b = 13$, and $a = 6$



$$\frac{13}{\sin 105°} = \frac{6}{\sin A}$$

$$13 \cdot \sin A = 6 \cdot \sin 105°$$

$$\sin A = 0.446$$

$$\angle A = \sin^{-1}(0.446)$$

$$\boxed{\angle A = 26.5°}$$

$$\angle C = 180° - 105° - 26.5°$$

$$\boxed{\angle C = 48.5°}$$

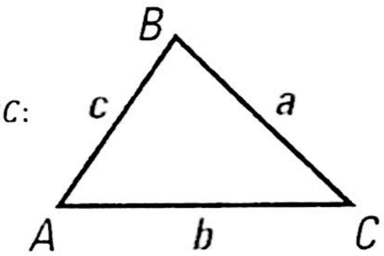
$$\frac{13}{\sin 105°} = \frac{c}{\sin 48.5°}$$

$$c \cdot \sin 105° = 13 \cdot \sin 48.5°$$

$$\boxed{c \approx 10.1}$$

Notes:

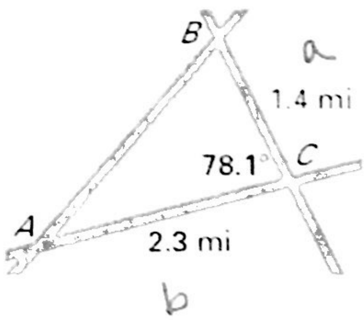
Area of a Triangle:



There are three ways we can find the area of ΔABC:

- Area = $\frac{1}{2} bc \sin A$
- Area = $\frac{1}{2} ac \sin B$
- Area = $\frac{1}{2} ab \sin C$

Example #5: A piece of land is bordered by three roads as shown. Find the area of the land.



$$\text{Area} = \frac{1}{2} ab \sin C$$

$$= \frac{1}{2}(1.4)(2.3)\sin 78.1°$$

$$\approx \boxed{1.58 \text{ mi}^2}$$