

## NOTES: Section 8.6 – Solve Rational Equations

Goals: #1 - I can solve a rational equation by cross multiplying.

#2 - I can solve a rational equation by using the least common denominator (LCD).

*Homework: Lesson 8.6 Worksheet*



Warm Up: Perform the indicated operation and simplify.

$$1. \frac{5}{x^2-1} + \frac{2x}{x^2+5x-6}$$

$$\text{LCD: } (x+1)(x-1)(x+6)$$

$$\frac{5(x+6)}{(x+1)(x-1)(x+6)} + \frac{2x(x+1)}{(x+1)(x-1)(x+6)}$$

$$\frac{5x+30}{(x+1)(x-1)(x+6)} + \frac{2x^2+2x}{(x+1)(x-1)(x+6)}$$

$$\boxed{\frac{2x^2+7x+30}{(x+1)(x-1)(x+6)}}$$

$$2. \frac{x-4}{3x-15} - \frac{x+1}{x^2-3x-10}$$

$$\text{LCD: } 3(x-5)(x+2)$$

$$\frac{(x-4)(x+2)}{3(x-5)(x+2)} - \frac{3(x+1)}{3(x-5)(x+2)}$$

$$\frac{x^2-2x-8}{3(x-5)(x+2)} - \frac{3x+3}{3(x-5)(x+2)}$$

$$\boxed{\frac{x^2-5x-11}{3(x-5)(x+2)}}$$

Exploration #1: Work with a partner and solve the following equations.

$$1. \frac{4}{x} = \frac{6}{15}$$

$$6x = 60$$

$$\boxed{x=10}$$

$$2. x(x-4) = 3(x-4) + x$$

$$x^2 - 4x = 3x - 12 + x$$

$$x^2 - 4x = 4x - 12$$

$$x^2 - 8x + 12 = 0$$

$$(x-6)(x-2) = 0$$

$$\boxed{x=6} \quad \boxed{x=2}$$

Notes:

When each side of the equation is a single rational expression, we can use cross multiplication to solve the rational equation.

Examples:

$$\frac{4}{x} = \frac{6}{15}$$

$$\frac{3}{x+1} = \frac{9}{4x+5}$$

Example #1: Solve the equation by cross multiplying. Check for extraneous solutions.

$$1. \frac{3}{x+1} \neq \frac{9}{4x+5}$$

$$9(x+1) = 3(4x+5)$$

$$9x + 9 = 12x + 15$$

$$-3x = 6$$

$$\boxed{x = -2}$$

check:

$$-1 \checkmark$$

$$-3 \checkmark$$

$$2. \frac{1}{2x+5} = \frac{x}{11x+8}$$

$$x(2x+5) = 11x+8$$

$$2x^2 + 5x = 11x + 8$$

$$2x^2 - 6x - 8 = 0$$

$$2(x^2 - 3x - 4) = 0$$

$$(x-4)(x+1) = 0$$

$$\boxed{x=4} \quad \boxed{x=-1}$$

check:

$$13 \checkmark$$

$$52 \checkmark$$

$$3 \checkmark$$

$$-3 \checkmark$$

You practice: Solve the equation by cross multiplying. Check for extraneous solutions.

$$1. \frac{3}{5x} = \frac{2}{x-7}$$

$$3(x-7) = 10x$$

$$3x - 21 = 10x$$

$$-21 = 7x$$

$$\boxed{x = -3}$$

check:

$$-15 \checkmark$$

$$-10 \checkmark$$

$$2. \frac{3}{x+2} = \frac{x-3}{2x+4}$$

$$(x+2)(x-3) = 3(2x+4)$$

$$x^2 - x - 6 = 6x + 12$$

$$x^2 - 7x - 18 = 0$$

$$(x-9)(x+2) = 0$$

$$\boxed{x=9}$$

$$\boxed{\cancel{x=-2}}$$

extraneous

check:

$$11 \checkmark$$

$$22 \checkmark$$

$$0 \times$$

Notes:

When a rational equation is NOT a proportion, we solve it by multiplying each side of the equation by the LCD (least common denominator) of each expression.

Example #2: Solve the equation by using the LCD. Check for extraneous solutions.

$$3x \cdot 1 \cdot \left( \frac{8}{x} + \frac{11}{3} \right) = \left( \frac{-14}{x} \right) 3x$$

LCD:  $3x$

$$\frac{8}{x} \cdot \frac{3x}{1} + \frac{11}{3} \cdot \frac{3x}{1} = \frac{-14}{x} \cdot \frac{3x}{1}$$

$$24 + 11x = -42$$

$$11x = -66$$

$$\boxed{x = -6}$$

check:  
-6 ✓

$$(x+3)(x-4) \cdot 2 \cdot \left( \frac{3x-5}{x+3} \right) = \left( \frac{2}{1} + \frac{8}{(x+3)(x-4)} \right) (x+3)(x-4)$$

LCD:  $(x+3)(x-4)$

$$\frac{3x-5}{x+3} \cdot \frac{(x+3)(x-4)}{1} = \frac{2}{1} \cdot \frac{(x+3)(x-4)}{1} + \frac{8}{(x+3)(x-4)} \cdot \frac{(x+3)(x-4)}{1}$$

$$3x^2 - 12x - 5x + 20 = 2(x^2 - x - 12) + 8$$

$$3x^2 - 17x + 20 = 2x^2 - 2x - 24 + 8$$

$$3x^2 - 17x + 20 = 2x^2 - 2x - 16$$

$$x^2 - 15x + 36 = 0$$

$$(x-12)(x-3) = 0$$

$$\boxed{x = 12} \quad \boxed{x = 3}$$

check:  
15 ✓ 6 ✓  
120 ✓ -6 ✓

You practice: Solve the equation by using the LCD. Check for extraneous solutions.

$$4x \cdot 1 \cdot \left( \frac{11}{4} - \frac{3}{x} \right) = \left( \frac{5}{2x} \right) 4x$$

LCD:  $4x$

$$\frac{11}{4} \cdot \frac{4x}{1} - \frac{3}{x} \cdot \frac{4x}{1} = \frac{5}{2x} \cdot \frac{4x}{1}$$

$$11x - 12 = 10$$

$$11x = 22$$

$$\boxed{x = 2}$$

check:  
2 ✓  
4 ✓

$$x(x+2) \cdot 2 \cdot \left( 1 + \frac{4}{x+2} \right) = \left( \frac{9}{x} \right) x(x+2)$$

LCD:  $x(x+2)$

$$1 \cdot \frac{x(x+2)}{1} + \frac{4}{x+2} \cdot \frac{x(x+2)}{1} = \frac{9}{x} \cdot \frac{x(x+2)}{1}$$

$$x^2 + 2x + 4x = 9x + 18$$

$$x^2 + 6x = 9x + 18$$

$$x^2 - 3x - 18 = 0$$

$$(x-6)(x+3) = 0$$

$$\boxed{x = 6} \quad \boxed{x = -3}$$

check:  
8 ✓ -1 ✓  
6 ✓ -3 ✓

Example #3: Solve the equation by using the LCD. Check for extraneous solutions.

$$1. \left( \frac{6x^2}{x^2-16} - \frac{3x}{x+4} \right) = \left( \frac{4}{x-4} \right) \quad (x+4)(x-4)$$

LCD:  $(x+4)(x-4)$

$$\frac{6x^2}{(x+4)(x-4)} \cdot \frac{(x+4)(x-4)}{1} - \frac{3x}{x+4} \cdot \frac{(x+4)(x-4)}{1} = \frac{4}{x-4} \cdot (x+4)(x-4)$$

$$6x^2 - (3x^2 - 12x) = 4x + 16$$

$$6x^2 - 3x^2 + 12x = 4x + 16$$

$3 \cdot -16 = -48$   
 $\wedge$   
 $12 \cdot -4$

$$3x^2 + 8x - 16 = 0$$

$$3x^2 + 12x \mid -4x - 16 = 0$$

$$3x(x+4) - 4(x+4) = 0$$

$(x+4)(3x-4) = 0$   
 extraneous  $\boxed{x = -4}$   $\boxed{x = 4/3}$

check:  
 $0 \times$   $-14.2 \checkmark$   
 $0 \times$   $5.3 \checkmark$   
 $0 \times$   $-2.6 \checkmark$

You practice: Solve the equation by using the LCD. Check for extraneous solutions.

$$1. \left( \frac{8x^2}{x^2-9} - \frac{4x}{x+3} \right) = \left( \frac{2}{x-3} \right) \quad (x+3)(x-3)$$

LCD:  $(x+3)(x-3)$

$$\frac{8x^2}{(x+3)(x-3)} \cdot \frac{(x+3)(x-3)}{1} - \frac{4x}{x+3} \cdot \frac{(x+3)(x-3)}{1} = \frac{2}{x-3} \cdot (x+3)(x-3)$$

$$8x^2 - (4x^2 - 12x) = 2x + 6$$

$$8x^2 - 4x^2 + 12x = 2x + 6$$

$$4x^2 + 10x - 6 = 0$$

$$2x^2 + 5x - 3 = 0$$

$$2x^2 + 6x \mid -1x - 3 = 0$$

$$2x(x+3) - 1(x+3) = 0$$

$$(x+3)(2x-1) = 0$$

$2 \cdot -3 = -6$   
 $\wedge$   
 $6 \cdot -1$

extraneous  $\boxed{x = 3}$   $x = 1/2$

check:  
 $0 \times$   $-8.8 \checkmark$   
 $0 \times$   $3.5 \checkmark$   
 $0 \times$   $-2.5 \checkmark$