Name: KEY Hour: ____________ Date: ____________

NOTES: Section 6.4 – Use Inverse Functions

Goals: #1 - I can find the inverse of a linear function.

#2 - I can verify that two functions are inverses of each other.

#3 - I can find the inverse of a power function.

#4 - I can graph the inverse of a function and determine if the inverse is a function.

Homework: Lesson 6.4 Worksheet

Warm Up:

1. Let $f(x) = 5x^3 - 2x$ and $g(x) = 3x^3$. Perform the indicated operation and state the domain.

a. $g(x) - f(x)$

$(3x^3) - (5x^3 - 2x)$

$-2x^3 + 2x$

answer: $\boxed{-2x^3 + 2x}$

domain: $\mathbb{R}$

b. $\dfrac{f(x)}{g(x)}$

$\dfrac{(5x^3 - 2x)}{(3x^3)}$

$\dfrac{5x^2 - 2}{3x^2}$

answer: $\boxed{\dfrac{5x^2 - 2}{3x^2}}$

domain: $\mathbb{R}, x \neq 0$

2. Let $f(x) = 4x^{-1}$ and $g(x) = 5x - 2$. Perform the indicated operation and state the domain.

a. $f(g(x))$

$f(5x-2) = 4(5x-2)^{-1}$

$\dfrac{4}{5x-2}$

answer: $\boxed{\dfrac{4}{5x-2}}$

domain: $\mathbb{R}, x \neq 2/5$

$5x - 2 = 0$

$5x = 2$

$x = 2/5$

b. $f(f(x))$

$f(4x^{-1}) = 4(4x^{-1})^{-1}$

$= 4(4^{-1}x)$

$= \dfrac{4x}{4}$

$= x$

answer: $\boxed{x}$

domain: $\mathbb{R}$

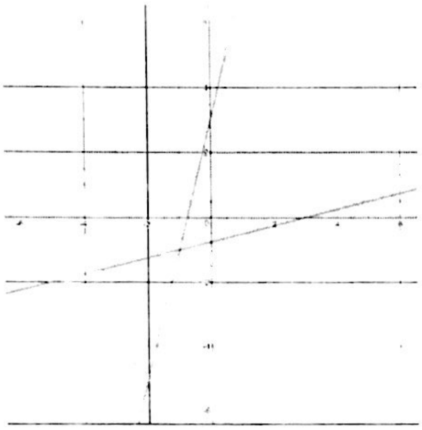
Name:________________ Hour: ________ Date: ________

Exploration #1: Work with a partner and answer the following questions.

1. Each pair of function are *inverses* of each other. Look at the graphs of the following functions. What do you notice?

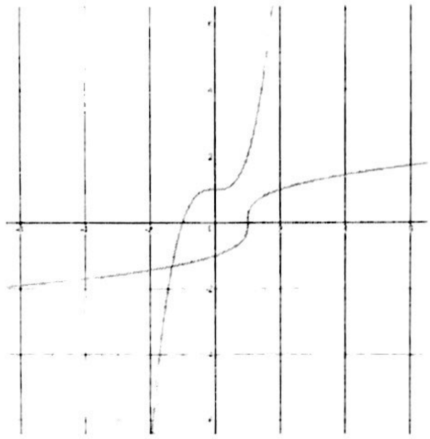
a. $f(x) = 4x + 3$

$g(x) = \frac{x-3}{4}$



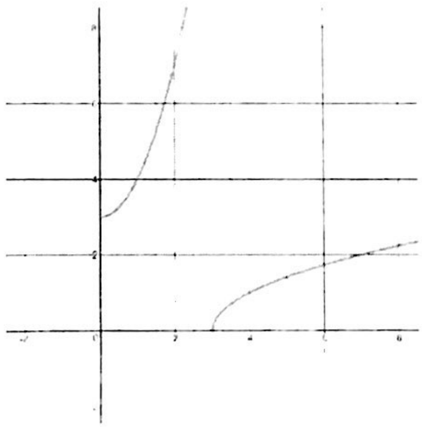
b. $f(x) = x^3 + 1$

$g(x) = \sqrt[3]{x-1}$



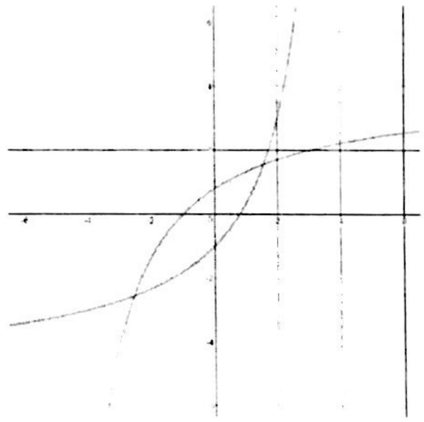
c. $f(x) = \sqrt{x-3}$

$g(x) = x^2 + 3, x \geq 0$



d. $f(x) = \frac{4x+4}{x+5}$

$g(x) = \frac{4-5x}{x-4}$



Notes: (y = x)

An inverse function is a reflection of the graph of the original relation. It interchanges the input and output values.

Meaning, the domain and range are also interchanged.

Name:________________________ Hour: __________ Date: __________

Example #1: Find an equation for the inverse of the relation.

1. $y = 3x - 5$

$x = 3y - 5$
$+5 \quad +5$
$\frac{x+5}{3} = \frac{3y}{3}$
$\boxed{y = \frac{1}{3}x + \frac{5}{3}}$

2. $y = -\frac{1}{3}x^3$

$-3(x) = (-\frac{1}{3}y^3) - 3$
$-3x = y^3$
$\sqrt[3]{-3x} = \sqrt[3]{y^3}$
$\boxed{y = \sqrt[3]{-3x}}$

You practice: Find an equation for the inverse of the relation.

1. $y = x^2 + 1$

$x = y^2 + 1$
$-1 \quad -1$
$x - 1 = y^2$
$\sqrt{x-1} = \sqrt{y^2}$
$\boxed{y = \sqrt{x-1}}$

2. $y = 4x + 2$

$x = 4y + 2$
$-2 \quad -2$
$\frac{x-2}{4} = \frac{4y}{4}$
$\boxed{y = \frac{1}{4}x - \frac{1}{2}}$

Notes:

Functions f and g are inverses of each other provided:

$$f(g(x)) = x \text{ and } g(f(x)) = x$$

Example #2: Verify that f and g are inverse functions.

1. $f(x) = 3x - 5;\ g(x) = \frac{1}{3}x + \frac{5}{3}$

$f(g(x))$
$f(\frac{1}{3}x + \frac{5}{3})$
$3(\frac{1}{3}x + \frac{5}{3}) - 5$
$x + 5 - 5$
$\boxed{x}$

$g(f(x))$
$g(3x-5)$
$\frac{1}{3}(3x-5) + \frac{5}{3}$
$x - \frac{5}{3} + \frac{5}{3}$
$\boxed{x}$ ✓

2. $f(x) = 6x^2 + 1, x \geq 0;\ g(x) = \left(\frac{x-1}{6}\right)^{\cancel{2}\,1/2}$

$f(g(x))$
$f\left(\left(\frac{x-1}{6}\right)^2\right)$
$6\left(\left(\frac{x-1}{6}\right)^2\right)^{1/2} + 1$
$6\left(\frac{x-1}{6}\right) + 1$
$x - 1 + 1$
$\boxed{x}$

$g(f(x))$
$g(6x^2+1)$
$\left(\frac{(6x^2+1)-1}{6}\right)^2$
$\left(\frac{6x^2}{6}\right)^2$
$(x^2)^{\frac{1}{2}}$
$\boxed{x}$ ✓

Name:________________________ Hour: __________ Date: __________

You practice: Verify that f and g are inverse functions.

1. $f(x) = \frac{2}{5}x + \frac{1}{3},\ g(x) = \frac{5}{2}x - \frac{5}{6}$

$f(g(x))$

$f\left(\frac{5}{2}x - \frac{5}{6}\right)$

$\frac{2}{5}\left(\frac{5}{2}x - \frac{5}{6}\right) + \frac{1}{3}$

$x - \frac{10}{30} + \frac{1}{3}$

$\boxed{x}$

$g(f(x))$

$g\left(\frac{2}{5}x + \frac{1}{3}\right)$

$\frac{5}{2}\left(\frac{2}{5}x + \frac{1}{3}\right) - \frac{5}{6}$

$x + \frac{5}{6} - \frac{5}{6}$

$\boxed{x}$ ✓

2. $f(x) = 6x^3,\ g(x) = \sqrt[3]{\frac{x}{6}}$

$f(g(x))$

$f\left(\sqrt[3]{\frac{x}{6}}\right)$

$6\left(\sqrt[3]{\frac{x}{6}}\right)^3$

$6\left(\frac{x}{6}\right)$

$\boxed{x}$

$g(f(x))$

$g(6x^3)$

$\sqrt[3]{\frac{6x^3}{6}}$

$\sqrt[3]{x^3}$

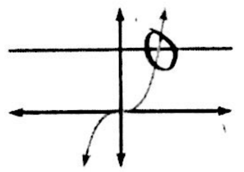
$\boxed{x}$ ✓

Notes:

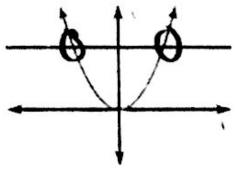
Recall, to determine if a graph is a function, we use the VLT (vertical line test).

To determine whether the inverse of a function is a function, we apply the Horizontal Line Test. (HLT)

Inverse is a function



Inverse is not a function



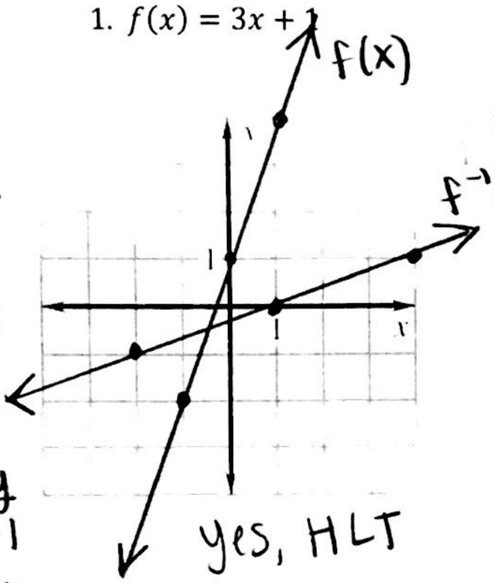
Name: KEY Hour: ____________ Date: ____________

Example #3: Graph the function f. Use the horizontal line test to determine whether the inverse of f is a function. Then graph the inverse of f.

1. $f(x) = 3x + 1$

f

x	y
-1	-2
0	1
1	4



f^{-1}

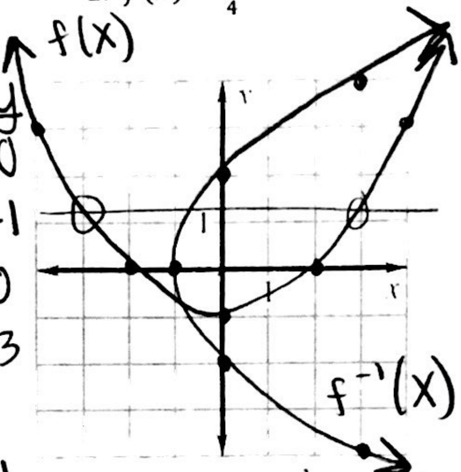
x	y
-2	-1
1	0
4	1

yes, HLT

2. $f(x) = \frac{1}{4}x^2 - 1$

f

x	y
-2	0
0	-1
2	0
4	3



f^{-1}

x	y
0	-2
-1	0
0	2
3	4

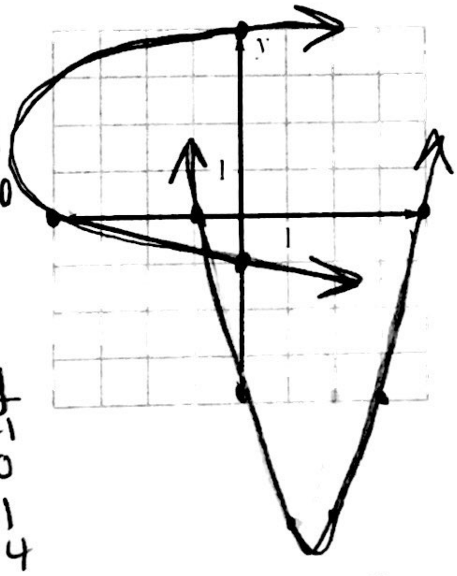
Inverse is NOT a function, HLT

You practice: Graph the function f. Use the horizontal line test to determine whether the inverse of f is a function. Then graph the inverse of f.

1. $f(x) = (x - 4)(x + 1)$

f

x	y
-1	0
0	-4
1	-6
4	0



f^{-1}

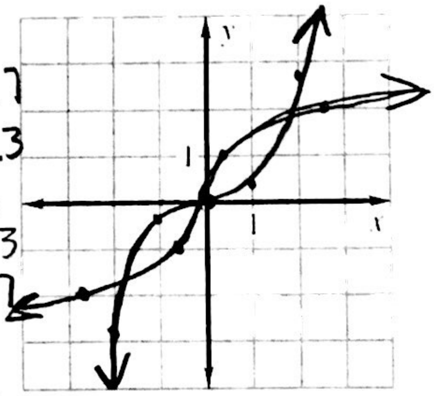
x	y
0	-1
-4	0
-6	1
0	4

Inverse is NOT a function, HLT

2. $f(x) = \frac{1}{3}x^3$

f

x	y
-2	-2.7
-1	-0.3
0	0
1	0.3
2	2.7



f^{-1}

x	y
-2.7	-2
-0.3	-1
0	0
0.3	1
2.7	2

Yes, HLT