Name: KEY Hour: ____________ Date: ____________

NOTES: Section 5.4 – Factor and Solve Polynomial Equations

Goals: #1 - I can factor a polynomial completely including methods for common monomials, difference of two squares, sum or difference of two cubes, factoring by grouping, the quadratic form.

#2 - I can recognize on my own which method of factoring is appropriate for a polynomial.

#3 - I can find the real number solutions of a polynomial. 😎 😬 😢

Homework: Lesson 5.4 Worksheet

Exploration #1: Work with a partner and answer the following questions.

1. Factor the following completely:

a. $2x^2 - 3x - 20$ $2(-20) = -40$, $(-8 + 5) = -3$

$2x^2 - 8x + 5x - 20$

$2x(x-4) + 5(x-4)$

$(x-4)(2x+5)$

b. $x^2 + 8x + 16$

$(x+4)(x+4)$

c. $9x^2 - 1$

$(4x-1)(4x+1)$

d. $8x^2 + 20x$

$4x(2x+5)$

Notes:

We can also factor polynomials with degree greater than 2!

Some of these polynomials can be factored completely using techniques we already learned!

Name:________________ Hour: ________ Date: ________

Example #1: Factor the polynomial completely.

1. $x^3 + 2x^2 - 15x$

$x(x^2 + 2x - 15)$

$x(x+5)(x-3)$

2. $2y^5 - 18y^3$

$2y^3(y^2 - 9)$

$2y^3(y-3)(y+3)$

You practice: Factor the polynomial completely.

2. $3x^3 + 30x^2 + 75x$

$3x(x^2 + 10x + 25)$

$3x(x+5)(x+5)$

$3x(x+5)^2$

2. $5g^5 - 80g^3$

$5g^3(g^2 - 16)$

$5g^3(g-4)(g+4)$

Notes:

There are special factoring patterns we can look for!

Sum of Two Cubes

$a^3 + b^3 = (a+b)(a^2 - ab + b^2)$

Ex: $a^3 + 8 = (a+2)(a^2 - 2a + 4)$

Difference of Two Cubes

$a^3 - b^3 = (a-b)(a^2 + ab + b^2)$

Ex: $a^3 - 8 = (a-2)(a^2 + 2a + 4)$

Example #2: Factor the polynomial completely.

1. $x^3 + 64$

$(x)^3 + (4)^3$

$a^3 + b^3 = (a+b)(a^2 - ab + b^2)$

$(x+4)(x^2 - 4x + 16)$

2. $16z^5 - 250z^2$

$(2z)^3 \quad (5)^3$

$2z^2(8z^3 - 125)$

$a^3 - b^3 = (a-b)(a^2 + ab + b^2)$

$2z^2(2z-5)(4z^2 + 10z + 25)$

Name:____________ Hour: ______ Date: ______

You practice: Factor the polynomial completely.

1. $w^3 - 27$

$(w)^3 \quad (3)^3$

$a^3 - b^3 = (a-b)(a^2+ab+b^2)$

$(w-3)(w^2+3w+9)$

2. $16b^5 + 686b^2$

$2b^2(8b^3+343)$

$(2b)^3 \quad (7)^3$

$a^3+b^3 = (a+b)(a^2-ab+b^2)$

$2b^2(2b+7)(4b^2-14b+49)$

Notes:

When a polynomial has 4 terms, we factor by grouping

Example #3: Factor the polynomial completely.

1. $x^3 - 3x^2 - 16x + 48$

$x^2(x-3) - 16(x-3)$

$(x-3)(x^2-16)$

$(x-3)(x-4)(x+4)$

2. $x^3 + 7x^2 - 9x - 63$

$x^2(x+7) - 9(x+7)$

$(x+7)(x^2-9)$

$(x+7)(x-3)(x+3)$

Notes:

Another pattern we can look for, is if a polynomial is in quadratic form

An expression in the form $au^2 + bu + c$ is in quadratic form.

Examples:

$2x^2 + x - 7$

$p^6 + 5p^3 + 6$

↑ middle term is ½ of degree

Name:______________________ Hour: __________ Date: __________

Example #4: Factor the polynomial completely.

1. $16x^4 - 81$

$(4x^2)^2 - (9)^2$

$(4x^2-9)(4x^2+9)$

$(2x)^2 \quad (3)^2$

$\boxed{(2x-3)(2x+3)(4x^2+9)}$

2. $2p^8 + 10p^5 + 12p^2$

$2p^2(p^6+5p^3+6)$

$\boxed{2p^2(p^3+3)(p^3+2)}$

You practice: Factor the polynomial completely.

1. $16g^4 - 625$

$(4g^2)^2 \quad (25)^2$

$(4g^2-25)(4g^2+25)$

$(2g)^2 \quad (5)^2$

$\boxed{(2g-5)(2g+5)(4g^2+25)}$

2. $4t^6 - 20t^4 + 24t^2$

$4t^2(t^4-5t^2+6)$

$\boxed{4t^2(t^2-3)(t^2-2)}$

Notes:

We can still use factoring to solve certain polynomial equations.

We set the polynomial equation equal to 0 and use the ZPP.

- **Zero Product Property:**

$a \cdot b = 0, \quad a=0 \quad b=0$

Example #5: Solve the following equations.

1. $3x^5 + 15x = 18x^3$

$3x^5 - 18x^3 + 15x = 0$

$3x(x^4-6x^2+5)=0$

$3x(x^2-5)(x^2-1)=0$

$3x(x^2-5)(x+1)(x-1)=0$

$3x=0$ → $\boxed{x=0}$

$x^2-5=0$, $x^2=5$ → $\boxed{x=\pm\sqrt{5}}$

$x+1=0$ → $\boxed{x=-1}$

$x-1=0$ → $\boxed{x=1}$

2. $-27x^3 + 15x^2 = -6x^4$

$6x^4-27x^3+15x^2=0$

$x^2(6x^2-27x+15)=0$

$x^2=0$ → $\boxed{x=0}$

$6x^2-27x+15=0$

$x=\frac{27\pm\sqrt{(-27)^2-4(6)(15)}}{2(6)}$

$x=\frac{27\pm\sqrt{369}}{12}$

$x=\frac{27\pm3\sqrt{41}}{12}$

$\boxed{x=\frac{9\pm\sqrt{41}}{4}}$

Name: ______________________ Hour: __________ Date: __________

You practice: Solve the following equations.

1. $4x^5 - 40x^3 + 36x = 0$

$4x(x^4 - 10x^2 + 9) = 0$

$4x(x^2 - 9)(x^2 - 1) = 0$

$4x(x-3)(x+3)(x-1)(x+1) = 0$

$4x = 0 \quad x-3=0 \quad x+3=0 \quad x-1=0 \quad x+1=0$

$\boxed{x=0} \quad \boxed{x=3} \quad \boxed{x=-3} \quad \boxed{x=1} \quad \boxed{x=-1}$

2. $2x^5 + 24x = 14x^3$

$2x^5 - 14x^3 + 24x = 0$

$2x(x^4 - 7x^2 + 12) = 0$

$2x(x^2-3)(x^2-4) = 0$

$2x(x^2-3)(x+2)(x-2) = 0$

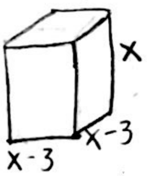
$2x = 0 \quad x^2-3=0 \quad x+2=0 \quad x-2=0$

$\boxed{x=0} \quad x^2 = 3 \quad \boxed{x=-2} \quad \boxed{x=2}$

$\boxed{x = \pm\sqrt{3}}$

Example #6: A catering company is designing a box for packing Christmas candies. The company would like the volume of the box to be 54 cubic inches and the bottom of the box to be a square.

a.) Suppose that the bottom of the box has a width that is 3 inches smaller than the height x of the box. Write a simplified polynomial equation, in standard form, for the volume of the box.



$V = x(x-3)(x-3)$

$= x(x^2 - 3x - 3x + 9)$

$= x(x^2 - 6x + 9)$

$= x^3 - 6x^2 + 9x$

a.) $\boxed{V = x^3 - 6x^2 + 9x}$

b.) Solve the equation from part (a).

$54 = x^3 - 6x^2 + 9x$

$0 = x^3 - 6x^2 + 9x - 54$

$0 = x^2(x-6) + 9(x-6)$

$0 = (x^2+9)(x-6)$

$x^2 + 9 = 0$

$x^2 = -9$

~~$x = \pm 3i$~~

$x - 6 = 0$

$\boxed{x=6}$

b.) $\boxed{x=6}$

c.) What are the dimensions of the box?

l: $x-3$ $6-3=3$

w: $x-3$ $6-3=3$

h: x 6

c.)

length: 3 in

width: 3 in

height: 6 in