Chapter 5 (Part 2) Review Worksheet

Name: **KEY**

Divide using polynomial long division.

1.) $(x^2 + 5x - 14) \div (x - 2)$

$$
\begin{array}{r}
x + 7 \\
x-2 \overline{)\ x^2 + 5x - 14} \\
-\underline{x^2 - 2x} \quad \downarrow \\
7x - 14 \\
-\underline{7x - 14} \\
0
\end{array}
$$

$$\boxed{x + 7}$$

2.) $(6x^2 - 5x + 9) \div (2x - 1)$

$$
\begin{array}{r}
3x - 1 \\
2x-1 \overline{)\ 6x^2 - 5x + 9} \\
-\underline{6x^2 - 3x} \quad \downarrow \\
-2x + 9 \\
-\underline{2x + 1} \\
8
\end{array}
$$

$$\boxed{3x - 1 + \dfrac{8}{2x-1}}$$

3.) $(5x^4 + 2x^3 - 9x + 12) \div (x^2 - 3x + 4)$

$$
\begin{array}{r}
5x^2 + 17x + 31 \\
x^2-3x+4 \overline{)\ 5x^4 + 2x^3 + 0x^2 - 9x + 12} \\
\underline{5x^4 - 15x^3 + 20x^2} \quad \downarrow \\
17x^3 - 20x^2 - 9x \\
\underline{17x^2 - 51x^2 + 68x} \quad \downarrow \\
31x^2 - 77x + 12 \\
\underline{31x^2 - 93x + 124} \\
16x - 112
\end{array}
$$

$$\boxed{5x^2 + 17x + 31 + \dfrac{16x - 112}{x^2 - 3x + 4}}$$

4.) $(4x^4 + 5x - 4) \div (x^2 - 3x - 2)$

$$
\begin{array}{r}
4x^2 + 12x + 44 \\
x^2-3x-2 \overline{)\ 4x^4 + 0x^3 + 0x^2 + 5x - 4} \\
\underline{4x^4 - 12x^3 - 8x^2} \quad \downarrow \\
12x^3 + 8x^2 + 5x \\
\underline{12x^3 - 36x^2 - 24x} \quad \downarrow \\
44x^2 + 29x - 4 \\
\underline{44x^2 - 132x - 88} \\
161x + 84
\end{array}
$$

$$\boxed{4x^2 + 12x + 44 + \dfrac{161x + 84}{x^2 - 3x - 2}}$$

Divide using synthetic division.

5.) $(x^4 - 7x^2 + 9x - 10) \div (x - 2)$

$$
\begin{array}{r|rrrrr}
2 & 1 & 0 & -7 & 9 & -10 \\
  &   & 2 & 4 & -6 & 6 \\
\hline
  & 1 & 2 & -3 & 3 & \boxed{-4}\ R
\end{array}
$$

$$\boxed{x^3 + 2x^2 - 3x + 3 + \dfrac{-4}{x-2}}$$

6.) $(2x^2 - 11x^3 + 15x^2 + 6x - 18) \div (x - 3)$

$-11x^3 + 17x^2 + 6x - 18$

$$
\begin{array}{r|rrrr}
3 & -11 & 17 & 6 & -18 \\
  &   & -33 & -48 & -126 \\
\hline
  & -11 & -16 & -42 & \boxed{-144}\ R
\end{array}
$$

$$\boxed{-11x^2 - 16x - 42 + \dfrac{-144}{x-3}}$$

Given polynomial $f(x)$ and a factor of $f(x)$, factor $f(x)$ completely.

7.) $f(x) = x^3 - 3x^2 - 16x - 12$; $(x - 6)$

$$\begin{array}{r|rrrr} 6 & 1 & -3 & -16 & -12 \\ & \downarrow & 6 & 18 & 12 \\ \hline & 1 & 3 & 2 & 0 \end{array}$$

$(x^2 + 3x + 2)(x - 6)$

$\boxed{(x + 2)(x + 1)(x - 6)}$

8.) $f(x) = 3x^3 - 16x^2 - 103x + 36$; $(x + 4)$

$$\begin{array}{r|rrrr} -4 & 3 & -16 & -103 & 36 \\ & \downarrow & -12 & 112 & -36 \\ \hline & 3 & -28 & 9 & 0 \end{array}$$

$\begin{matrix} 27 \\ \diagup\!\diagdown \\ -27 \;\; -1 \end{matrix}$

$(3x^2 - 28x + 9)(x + 4)$
$(3x^2 - 27x + x + 9)(x + 4)$
$(3x(x - 9) - 1(x - 9))(x + 4)$
$\boxed{(3x - 1)(x - 9)(x + 4)}$

Given polynomial function f and a zero of f, find the other zeros of the function.

9.) $f(x) = 2x^3 + 3x^2 - 39x - 20$; zero: 4

$$\begin{array}{r|rrrr} 4 & 2 & 3 & -39 & -20 \\ & \downarrow & 8 & 44 & 20 \\ \hline & 2 & 11 & 5 & 0 \end{array}$$

$(2x^2 + 11x + 5)(x - 4) = 0$
$(2x^2 + 10x + x + 5)(x - 4) = 0$
$(2x(x + 5) + 1(x + 5))(x - 4) = 0$
$(2x + 1)(x + 5)(x - 4) = 0$
$\boxed{x = -\tfrac{1}{2}}$ $\boxed{x = -5}$ $\boxed{x = 4}$

10.) $f(x) = x^3 - 9x^2 - 5x + 45$; zero: 9

$$\begin{array}{r|rrrr} 9 & 1 & -9 & -5 & 45 \\ & \downarrow & 9 & 0 & -45 \\ \hline & 1 & 0 & -5 & 0 \end{array}$$

$(x^2 - 5)(x - 9) = 0$
$x^2 - 5 = 0$ $\boxed{x = 9}$
$x^2 = 5$
$\boxed{x = \pm\sqrt{5}}$

Find all **real** zeros of the function.

11.) $h(x) = x^3 + 4x^2 + x - 6$

$\pm 1, \pm 2, \pm 3, \pm 6$

$$\begin{array}{r|rrrr} 1 & 1 & 4 & 1 & -6 \\ & \downarrow & 1 & 5 & 6 \\ \hline & 1 & 5 & 6 & 0 \end{array}$$

$(x^2 + 5x + 6)(x - 1) = 0$
$(x + 3)(x + 2)(x - 1) = 0$
$\boxed{x = -3}$ $\boxed{x = -2}$ $\boxed{x = 1}$

12.) $g(x) = x^3 - 5x^2 - 18x + 72$

$\pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 8, \pm 9, \pm 12, \pm 18, \pm 24, \pm 36, \pm 72$

$$\begin{array}{r|rrrr} 3 & 1 & -5 & -18 & 72 \\ & \downarrow & 3 & -6 & -72 \\ \hline & 1 & -2 & -24 & 0 \end{array}$$

$(x^2 - 2x - 24)(x - 3) = 0$
$(x - 6)(x + 4)(x - 3) = 0$
$\boxed{x = 6}$ $\boxed{x = -4}$ $\boxed{x = 3}$

Find all __real__ zeros of the function.

13.) $f(x) = 2x^3 + 4x^2 - 2x - 4$

$\pm 1, \pm 2, \pm 4, \pm \frac{1}{2}$

$$
\begin{array}{r|rrrr}
1 & 2 & 4 & -2 & -4 \\
& \downarrow & 2 & 6 & 4 \\
\hline
& 2 & 6 & 4 & \boxed{0}
\end{array}
$$

$(2x^2 + 6x + 4)(x-1) = 0$
$(2x^2 + 4x + 2x + 4)(x-1) = 0$
$(2x(x+2) + 2(x+2))(x-1) = 0$
$(2x+2)(x+2)(x-1) = 0$

$\boxed{x = -1}$ $\boxed{x = -2}$ $\boxed{x = 1}$

14.) $g(x) = 2x^3 - 5x^2 - 14x + 8$

$\pm 1, \pm 2, \pm 4, \pm 8, \pm \frac{1}{2}$

$$
\begin{array}{r|rrrr}
-2 & 2 & -5 & -14 & 8 \\
& \downarrow & -4 & 18 & -8 \\
\hline
& 2 & -9 & 4 & \boxed{0}
\end{array}
$$

$(2x^2 - 9x + 4)(x+2) = 0$
$(2x^2 - 8x + x + 4)(x+2) = 0$
$(2x(x-4) - 1(x-4))(x+2) = 0$
$(2x-1)(x-4)(x+2) = 0$

$\boxed{x = \frac{1}{2}}$ $\boxed{x = 4}$ $\boxed{x = -2}$

Find __all__ zeros of the polynomial function.

15.) $f(x) = x^4 + 4x^3 + 7x^2 + 16x + 12$

$\pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 12$

$$
\begin{array}{r|rrrrr}
-1 & 1 & 4 & 7 & 16 & 12 \\
& \downarrow & -1 & -3 & -4 & -12 \\
\hline
& 1 & 3 & 4 & 12 & \boxed{0}
\end{array}
$$

$x^3 + 3x^2 + 4x + 12$

$$
\begin{array}{r|rrrr}
-3 & 1 & 3 & 4 & 12 \\
& \downarrow & -3 & 0 & -12 \\
\hline
& 1 & 0 & 4 & \boxed{0}
\end{array}
$$

$x^2 + 4$

$(x+1)(x+3)(x^2+4) = 0$

$\boxed{x = -1}$ $\boxed{x = -3}$

$x^2 + 4 = 0$
$x^2 = -4$

$\boxed{x = \pm 2i}$

16.) $g(x) = x^4 + 5x^3 - 7x^2 - 29x + 30$

$\pm 1, \pm 2, \pm 3, \pm 5, \pm 6, \pm 10, \pm 15, \pm 30$

$$
\begin{array}{r|rrrrr}
1 & 1 & 5 & -7 & -29 & 30 \\
& \downarrow & 1 & 6 & -1 & -30 \\
\hline
& 1 & 6 & -1 & -30 & \boxed{0}
\end{array}
$$

$0 = (x-1)(x-2)(x^2+8x+15)$
$0 = (x-1)(x-2)(x+3)(x+5)$

$x^3 + 6x^2 - x - 30$

$$
\begin{array}{r|rrrr}
2 & 1 & 6 & -1 & -30 \\
& \downarrow & 2 & 16 & 30 \\
\hline
& 1 & 8 & 15 & \boxed{0}
\end{array}
$$

$x^2 + 8x + 15$

$\boxed{x = 1}$ $\boxed{x = 2}$

$\boxed{x = -3}$ $\boxed{x = -5}$

Write a polynomial function _f_ of least degree that has rational coefficients, a leading coefficient of 1, and the given zeros.

17.) $-3, -1, -2i$

$(x+3)(x+1)(x-2i)(x+2i)$
$(x^2 + x + 3x + 3)(x^2 + 2ix - 2ix - 4i^2)$
$(x^2 + 4x + 3)(x^2 + 4)$
$x^2(x^2 + 4x + 3) + 4(x^2 + 4x + 3)$
$x^4 + 4x^3 + 3x^2 + 4x^2 + 16x + 12$

$\boxed{x^4 + 4x^3 + 7x^2 + 16x + 12}$

18.) $3, 2 + \sqrt{3}$

$(x-3)(x - (2+\sqrt{3}))(x - (2 - \sqrt{3}))$
$(x-3)((x-2) - \sqrt{3})((x-2) + \sqrt{3})$
$(x-3)((x-2)^2 + \sqrt{3}(x-2) - \sqrt{3}(x-2) - 3)$
$(x-3)(x^2 - 2x - 2x + 4 - 3)$
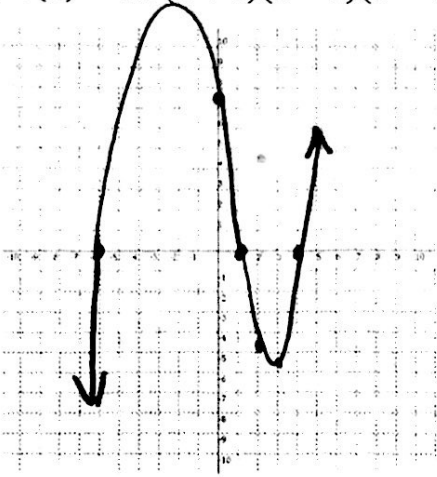$(x-3)(x^2 - 4x + 1)$
$x(x^2 - 4x + 1) - 3(x^2 - 4x + 1)$
$x^3 - 4x^2 + x - 3x^2 + 12x - 3$

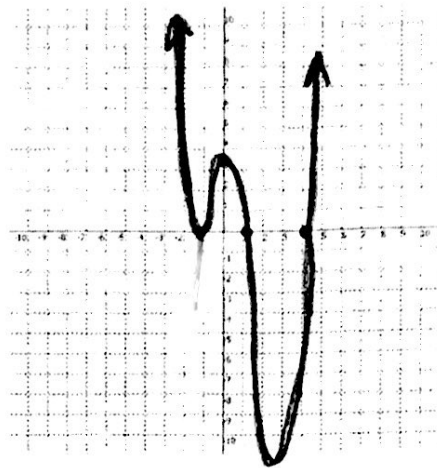$\boxed{x^3 - 7x^2 + 13x - 3}$

19.) $h(x) = 0.3(x+6)(x-1)(x-4)$

x-intercept(s): $(-6,0)(1,0)(4,0)$

y-intercept: $(0, 7.2)$

x	-7	-5	-4	-1	2	3	5
y	-26.4	162	24	15	-4.8	-5.4	132

20.) $f(x) = \frac{5}{6}(x+1)^2(x-1)(x-4)$

x-intercept(s): $(-1,0)(1,0)(4,0)$

y-intercept: $(0, 3.3)$

x	-2	2	3		5		
y	15	-15	-26.7		120		

21.) $h(x) = x^3 - x^2 - 17x - 15$
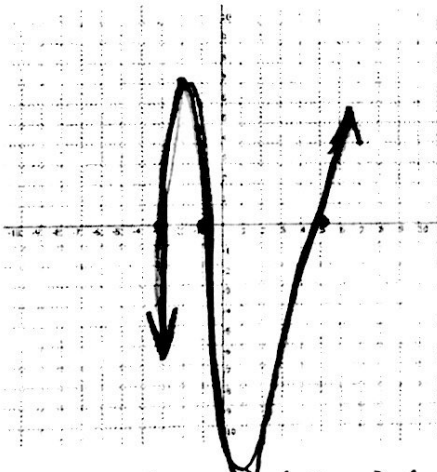
x-intercept(s): $(-1, 0)(5,0)(-3,0)$

y-intercept: $(0, -15)$

x	-4	-2	1	2	3	4	6
y	-27	7	-32	-45	-48	-35	63

$\pm1, \pm3, \pm5, \pm15 \qquad x^2 - 2x - 15$

$(x-5)(x+3)$

$$
\begin{array}{r|rrrr}
-1 & 1 & -1 & -17 & -15 \\
   &   & -1 & 2   & 15 \\
\hline
   & 1 & -2 & -15 & 0
\end{array}
$$

22.) $f(x) = x^3 - 4x^2 - 3x + 18$
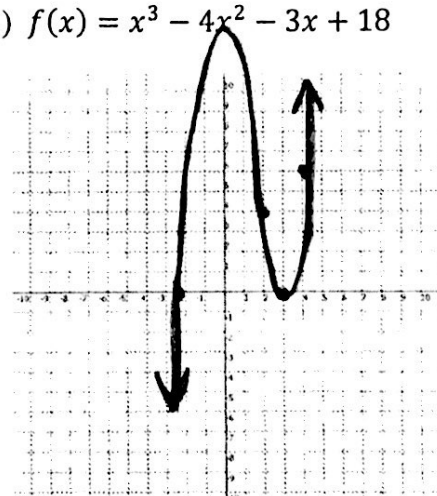
x-intercept(s): $(-2,0) \quad (3,0)$

y-intercept: $(0, 18)$

x	-3	-1	1	2	4	5	
y	-36	16	12	4	6	28	

$\pm1, \pm2, \pm3, \pm6, \pm9, \pm18 \qquad x^2 - 6x + 9$

$(x-3)(x-3)$

$$
\begin{array}{r|rrrr}
-2 & 1 & -4 & -3 & 18 \\
   &   & -2 & 12 & -18 \\
\hline
   & 1 & -6 & 9 & 0
\end{array}
$$

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