Name: LEY Hour: ____________ Date: ____________

NOTES: Section 4.8 – Use the Quadratic Formula and the Discriminant

Goals: #1 - I can use the quadratic formula to solve a quadratic equation. 😎 😬 😢

#2 - I can find the discriminant of a quadratic equation and use it to find the number and type of solutions.

Homework: Lesson 4.8 Worksheet

Warm Up:

Solve the equation by completing the square.

1. $x^2 - 14x + 9 = 0$ $\left(\frac{-14}{2}\right)^2 \to (-7)^2 \to 49$

$x^2 - 14x + \boxed{49} = -9 + \boxed{49}$

$(x-7)^2 = 40$

$x - 7 = \pm\sqrt{40} < \begin{matrix}\sqrt{4}\\ \sqrt{10}\end{matrix}$

$\boxed{x = 7 \pm 2\sqrt{10}}$

2. $3x^2 - 24x = -48$ $\left(\frac{-8}{2}\right)^2 \to (-4)^2 \to 16$

$\frac{3x^2 - 24x}{3} = \frac{-48}{3}$

$x^2 - 8x = -16$

$x^2 - 8x + 16 = 0$

$(x-4)^2 = 0$

$x - 4 = 0$

$\boxed{x = 4}$

Write the following quadratic function in vertex form. Then identify the vertex.

3. $y = 3x^2 + 24x + 40$ $\left(\frac{8}{2}\right)^2 \to (4)^2 \to 16$

$-40 \qquad -40$

$y - 40 = 3(x^2 + 8x)$

$\boxed{16} + \frac{y-40}{3} = x^2 + 8x + \boxed{16}$

$16 + \frac{y-40}{3} = (x+4)^2$

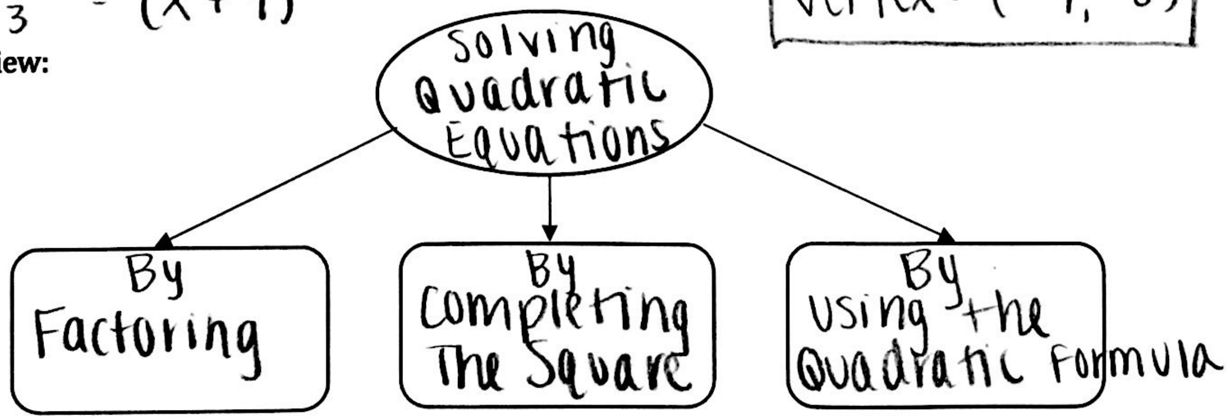
$\frac{y-40}{3} = (x+4)^2 - 16$

$y - 40 = 3(x+4)^2 - 48$

$\boxed{y = 3(x+4)^2 - 8}$

$\boxed{\text{vertex: } (-4, -8)}$

Review:



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Notes:

We can find the solutions to ANY quadratic equation by using the quadratic formula: $x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$

This formula can be derived from completing the square.

$ax^2+bx+c=0$

$ax^2+bx=-c$

$a(x^2+\frac{b}{a}x)=-c$

$x^2+\frac{b}{a}x=\frac{-c}{a}$

$x^2+\frac{b}{a}x+\frac{b^2}{4a^2}=\frac{-c}{a}+\frac{b^2}{4a^2}$

$(x+\frac{b}{2a})^2=\frac{-c(4a)}{4a^2}+\frac{b^2}{4a^2}$

$(x+\frac{b}{2a})^2=\frac{b^2-4ac}{4a^2}$

$x+\frac{b}{2a}=\pm\sqrt{\frac{b^2-4ac}{4a^2}}$

$x=\frac{-b}{2a}\pm\frac{\sqrt{b^2-4ac}}{2a}$

$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$

Example #1: Use the quadratic formula to solve the equation.

1. $x^2+3x=2$

$x^2+3x-2=0$

a b c

$x=\frac{-(3)\pm\sqrt{(3)^2-4(1)(-2)}}{2(1)}$

$x=\frac{-3\pm\sqrt{9+8}}{2}$

$\boxed{x=\frac{-3\pm\sqrt{17}}{2}}$

2. $25x^2-18x=12x-9$

$25x^2-30x+9=0$

a b c

$x=\frac{-(-30)\pm\sqrt{(-30)^2-4(25)(9)}}{2(25)}$

$x=\frac{30\pm\sqrt{900-900}}{50}$

$x=\frac{30\pm\sqrt{0}}{50}$

$x=\frac{30}{50}$

$\boxed{x=\frac{3}{5}}$

3. $-x^2+4x=5$

$-x^2+4x-5=0$

a b c

$x=\frac{-(4)\pm\sqrt{(4)^2-4(-1)(-5)}}{2(-1)}$

$x=\frac{-4\pm\sqrt{16-20}}{-2}$

$x=\frac{-4\pm\sqrt{-4}}{-2}$

$x=\frac{-4\pm 2i}{-2}$

$\boxed{x=2\pm i}$

You practice: Use the quadratic formula to solve the equation.

1. $4x^2-10x=2x-9$

$4x^2-12x+9=0$

a b c

$x=\frac{-(-12)\pm\sqrt{(-12)^2-4(4)(9)}}{2(4)}$

$x=\frac{12\pm\sqrt{144-144}}{8}$

$x=\frac{12\pm\sqrt{0}}{8}$

$x=\frac{12}{8}$

$\boxed{x=\frac{3}{2}}$

2. $7x-5x^2-4=2x+3$

$-5x^2+5x-7=0$

a b c

$x=\frac{-(5)\pm\sqrt{(5)^2-4(-5)(-7)}}{2(-5)}$

$x=\frac{-5\pm\sqrt{25-140}}{-10}$

$x=\frac{-5\pm\sqrt{-115}}{-10}$

$\boxed{x=\frac{5\pm i\sqrt{115}}{10}}$

CHALLENGE: What do you notice about the value the radical symbol in the last 5 examples?

$\sqrt{+\#} \rightarrow$ 2 real solutions $\quad \sqrt{0} \rightarrow$ 1 solution $\quad \sqrt{-\#} \rightarrow$ 2 imaginary solutions

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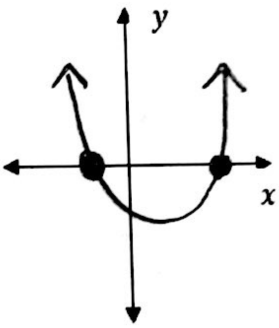
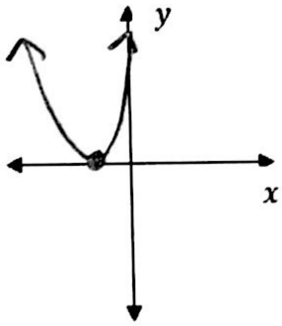
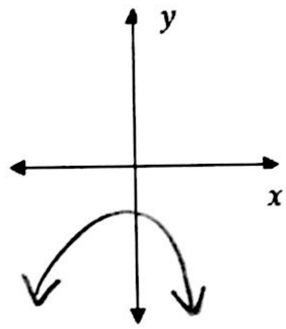
Notes:

In the quadratic formula, the expression b^2-4ac is called the discriminant of the quadratic equation $ax^2+bx+c=0$.

$$x = \frac{-b \pm \sqrt{\boxed{b^2 - 4ac}}}{2a} \rightarrow \text{discriminant}$$

We can use the discriminant of a quadratic equation to determine the equation's number and type of solutions.

Value of discriminant	positive $b^2-4ac>0$	zero $b^2-4ac=0$	negative $b^2-4ac<0$
Number and type of solutions	Two real solutions	One real solution	Two imaginary solutions
Graph of $y = ax^2 + bx + c$			

Example #2: Find the discriminant of the quadratic equation and give the number and types of solutions of the equation.

1. $\overset{a}{x^2} \overset{b}{- 8x} + \overset{c}{17} = 0$

b^2-4ac

$(-8)^2-4(1)(17)$

$64-68$

$\boxed{-4}$

$\boxed{\text{Two imaginary}}$

2. $2x^2 = 16x - 32$

$\underset{a}{2x^2}\underset{b}{-16x}+\underset{c}{32}=0$

b^2-4ac

$(-16)^2-4(2)(32)$

$256-256$

$\boxed{0}$

$\boxed{\text{One real}}$

3. $\overset{a}{x^2} \overset{b}{- 8x} + \overset{c}{15} = 0$

b^2-4ac

$(-8)^2-4(1)(15)$

$64-60$

$\boxed{4}$

$\boxed{\text{Two real}}$

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You practice: Find the discriminant of the quadratic equation and give the number and types of solutions of the equation.

1. $\overset{a}{3x^2} + \overset{b}{12x} + \overset{c}{12} = 0$

$b^2 - 4ac$

$(12)^2 - 4(3)(12)$

$144 - 144$

$\boxed{0}$

$\boxed{\text{one real}}$

2. $8x^2 = 9x - 11$

$8x^2 - 9x + 11 = 0$

a b c

$b^2 - 4ac$

$(-9)^2 - 4(8)(11)$

$81 - 352$

$\boxed{-271}$

$\boxed{\text{two imaginary}}$

3. $7x^2 - 2x = 5$

$7x^2 - 2x - 5 = 0$

a b c

$b^2 - 4ac$

$(-2)^2 - 4(7)(-5)$

$4 + 140$

$\boxed{144}$

$\boxed{\text{two real}}$

Notes:

In Section 4.5, the function $h = -16t^2 + h_0$ was used to model the height of a dropped object. For an object that is launched or thrown, an extra term v_0t must be added to the model to account for the object's initial velocity.

- **Object is *dropped*:** $h = -16t^2 + h_0$
- **Objected is *launched* or *thrown*:** $h = -16t^2 + v_0t + h_0$

The value of v_0 can be positive, negative, or zero depending on whether the object is launched upward, downward or parallel to the ground.

Example #3: A juggler tosses a ball into the air. The ball leaves the juggler's hand 4 feet above the ground and has an initial vertical velocity of 40 feet per second. The juggler catches the ball when it falls back to the height of 3 feet. How long is the ball in the air? initial height

$$h = -16t^2 + v_0t + h_0$$

$$3 = -16t^2 + 40t + 4$$

$$0 = -16t^2 + 40t + 1$$

$$t = \frac{-40 \pm \sqrt{(40)^2 - 4(-16)(1)}}{2(-16)}$$

$$t = \frac{-40 \pm \sqrt{1664}}{-32}$$

$$t \approx -0.025 \quad \text{or} \quad \boxed{t \approx 2.5 \text{ seconds}}$$