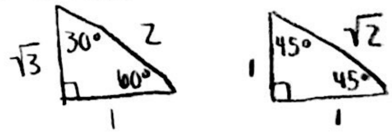
Review Lessons 13.1 – 13.3 Worksheet

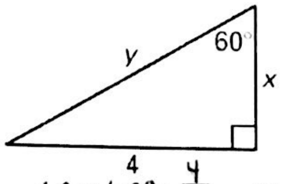
Name: KEY

NO CALCULATOR, NO UNIT CIRCLE



Find the exact values of *x* and *y*.

1.)



$\sin 60° = \frac{4}{y}$

$\frac{\sqrt{3}}{2} = \frac{4}{y}$

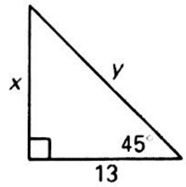
$\sqrt{3}\,y = 8$

$y = \frac{8}{\sqrt{3}} = \boxed{\frac{8\sqrt{3}}{3}}$ y

$\tan 60° = \frac{4}{x}$ $\quad \sqrt{3}x = 4$

$\frac{\sqrt{3}}{1} = \frac{4}{x}$ $\quad x = \frac{4}{\sqrt{3}} = \boxed{\frac{4\sqrt{3}}{3}}$ x

2.)



$\boxed{x = 13}$

$\cos 45° = \frac{13}{y}$

$\frac{\sqrt{2}}{2} = \frac{13}{y}$

$\sqrt{2}y = 26$

$y = \frac{26}{\sqrt{2}} = \frac{26\sqrt{2}}{2}$

$\boxed{y = 13\sqrt{2}}$

Evaluate the trigonometric function. Give an exact answer.

3.) $\tan \frac{\pi}{6} \to 30°$

$\tan \frac{\pi}{6} = \frac{1}{\sqrt{3}} = \boxed{\frac{\sqrt{3}}{3}}$

4.) $\csc \frac{\pi}{3} \to 60°$ $\quad \sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}$

$\csc \frac{\pi}{3} = \frac{2}{\sqrt{3}} = \boxed{\frac{2\sqrt{3}}{3}}$

5.) $\sin \frac{\pi}{4} \to 45°$

$\sin \frac{\pi}{4} = \frac{1}{\sqrt{2}} = \boxed{\frac{\sqrt{2}}{2}}$

6.) $\cos \frac{\pi}{3} \to 60°$

$\cos \frac{\pi}{3} = \boxed{\frac{1}{2}}$

7.) $\sec \frac{\pi}{4} \to 45°$ $\quad \cos\frac{\pi}{4} = \frac{1}{\sqrt{2}}$

$\sec \frac{\pi}{4} = \boxed{\sqrt{2}}$

8.) $\cot \frac{\pi}{6} \to 30°$ $\quad \tan\frac{\pi}{6} = \frac{1}{\sqrt{3}}$

$\cot \frac{\pi}{6} = \boxed{\sqrt{3}}$

Evaluate the six trigonometric functions of θ.

9.) $\theta = -270°$ $\quad (0,1)$

$\sin(-270°) = 1$ $\quad \csc(-270°) = 1$

$\cos(-270°) = 0$ $\quad \sec(-270°) = \frac{1}{0}$ UND

$\tan(-270°) = \frac{1}{0}$ UND $\quad \cot(-270°) = \frac{0}{1} = 0$

10.) $\theta = -\pi$ $\quad (-1,0)$

$\sin(-\pi) = 0$ $\quad \csc(-\pi) = \frac{1}{0}$ UND

$\cos(-\pi) = -1$ $\quad \sec(-\pi) = -1$

$\tan(-\pi) = \frac{0}{-1} = 0$ $\quad \cot(-\pi) = \frac{-1}{0}$ UND

NO CALCULATOR, MAY USE UNIT CIRCLE

Evaluate the function without using a calculator (i.e. ALL ANSWERS SHOULD BE EXACT, NO DECIMALS).

11.) $\sin(-120°)$

$\theta' = 60°$

y $\sin 60° = \frac{\sqrt{3}}{2}$

$\sin(-120°) = \boxed{-\frac{\sqrt{3}}{2}}$

12.) $\sec 45°$

$\frac{1}{x}$

$\sec 45° = \frac{1}{\frac{\sqrt{2}}{2}}$

$= 1 \cdot \frac{2}{\sqrt{2}}$

$= \frac{2}{\sqrt{2}} = \frac{2\sqrt{2}}{2}$

$= \boxed{\sqrt{2}}$

13.) $\cot(-390°)$ $\quad$ 30° $\quad \theta' = 30°$

$\frac{x}{y}$ $\quad \cot 30° = \frac{\frac{\sqrt{3}}{2}}{\frac{1}{2}}$

$= \frac{\sqrt{3}}{2} \cdot \frac{2}{1}$

$= \sqrt{3}$

$\cot(-390°) = \boxed{-\sqrt{3}}$

$\cos \frac{17\pi}{6}$

$\theta' = \frac{\pi}{6}$

$\cos \frac{\pi}{6} = \frac{\sqrt{3}}{2}$

$\cos \frac{17\pi}{6} = \boxed{-\frac{\sqrt{3}}{2}}$

15.) $\frac{1}{y}$ $\csc\left(-\frac{3\pi}{4}\right)$

$\theta' = \frac{\pi}{4}$

$\csc \frac{\pi}{4} = \frac{1}{\frac{\sqrt{2}}{2}}$

$= 1 \cdot \frac{2}{\sqrt{2}}$

$= \frac{2}{\sqrt{2}} = \frac{2\sqrt{2}}{2} = \sqrt{2}$

$\csc\left(-\frac{3\pi}{4}\right) = \boxed{-\sqrt{2}}$

16.) $\frac{y}{x}$ $\tan \frac{8\pi}{3}$

$\theta' = \frac{\pi}{3}$

$\tan \frac{\pi}{3} = \frac{\frac{\sqrt{3}}{2}}{\frac{1}{2}}$

$= \frac{\sqrt{3}}{2} \cdot \frac{2}{1}$

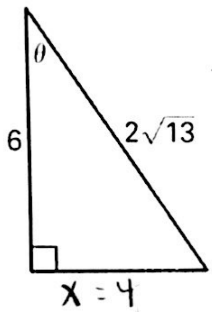
$= \sqrt{3}$

$\tan \frac{8\pi}{3} = \boxed{-\sqrt{3}}$

MAY USE CALCULATOR, MAY USE REFERENCE SHEET

Evaluate the six trigonometric functions of the angle θ.

17.)



$6^2 + x^2 = (2\sqrt{13})^2$

$36 + x^2 = 4(13)$

$x^2 = 16$

$x = 4$

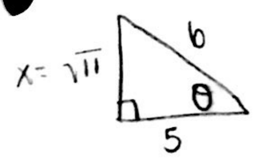
$\sin\theta = \frac{4}{2\sqrt{13}} = \frac{4\sqrt{13}}{26} = \boxed{\frac{2\sqrt{13}}{13}}$ $\csc\theta = \frac{2\sqrt{13}}{4} = \boxed{\frac{\sqrt{13}}{2}}$

$\cos\theta = \frac{6}{2\sqrt{13}} = \frac{6\sqrt{13}}{26} = \boxed{\frac{3\sqrt{13}}{3}}$ $\sec\theta = \frac{2\sqrt{13}}{6} = \boxed{\frac{\sqrt{13}}{3}}$

$\tan\theta = \frac{4}{6} = \boxed{\frac{2}{3}}$ $\cot\theta = \boxed{\frac{3}{2}}$

Let θ be an acute angle of a right triangle. Find the values of the other five trigonometric fucntions of θ.

18.) $\cos\theta = \frac{5}{6}$



$x^2 + 5^2 = 6^2$

$x^2 + 25 = 36$

$x^2 = 11$

$x = \sqrt{11}$

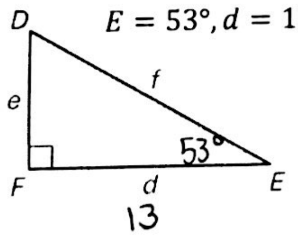
$\sin\theta = \boxed{\frac{\sqrt{11}}{6}}$ $\csc\theta = \frac{6}{\sqrt{11}} = \boxed{\frac{6\sqrt{11}}{11}}$

$\cos\theta = \boxed{\frac{5}{6}}$ $\sec\theta = \boxed{\frac{6}{5}}$

$\tan\theta = \boxed{\frac{\sqrt{11}}{5}}$ $\cot\theta = \frac{5}{\sqrt{11}} = \boxed{\frac{5\sqrt{11}}{11}}$

Solve ΔABC using the diagram and the given measurements. Round answers to the nearest tenth, when necessary.

19.) $E = 53°, d = 13$



$\angle D = 180° - 90° - 53°$

$\boxed{\angle D = 37°}$

$\tan 53° = \frac{e}{13}$

$e = 13 \tan 53°$

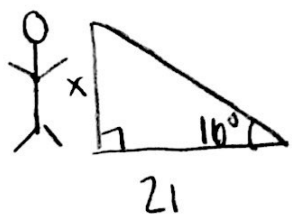
$\boxed{e \approx 17.3}$

$\cos 53° = \frac{13}{f}$

$f \cdot \cos 53° = 13$

$\boxed{f \approx 21.6}$

20.) A person casts a 21 foot shadow when the sun is at a 16° angle of elevation. What is the approximate height of the person?



$\tan 61° = \frac{x}{21}$

$x = 21 \cdot \tan 61°$

$x \approx \boxed{6 \text{ ft tall}}$

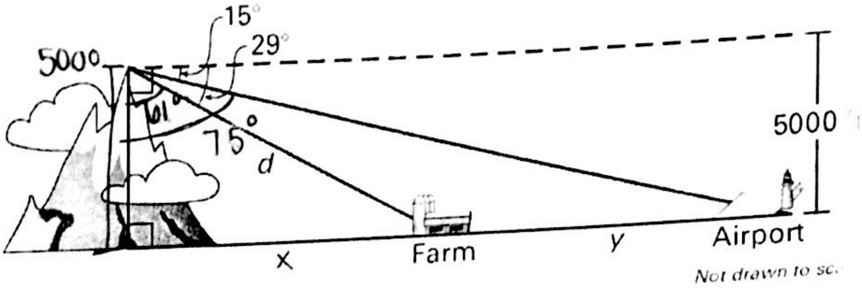
A hiker at the top of a mountain sees a farm and an airport in the distance.

a.) What is the distance d from the hiker to the farm?

$\cos 61^\circ = \frac{5000}{d}$

$d \cdot \cos 61^\circ = 5000$

$d \approx \boxed{10{,}313 \text{ ft}}$



b.) What is the distance y from the farm to the airport?

$\tan 61^\circ = \frac{x}{5000}$

$x = 5000 \cdot \tan 61^\circ$

$x \approx 9{,}020.23 \text{ ft}$

$\tan 75^\circ = \frac{9{,}020.23 \text{ ft} + y}{5000}$

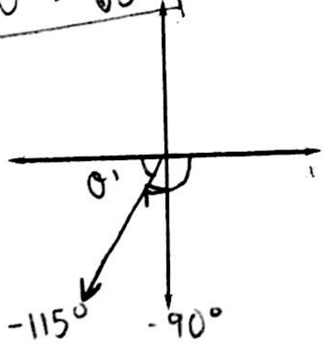
$5000 \tan 75^\circ = 9{,}020.23 + y$

$y \approx \boxed{9{,}6040 \text{ ft}}$

Sketch the angle. Then find its reference angle. Answer in the unit of the given angle.

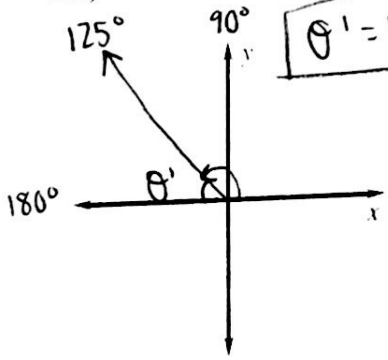
22.) $-115°$

$\boxed{\theta' = 65^\circ}$



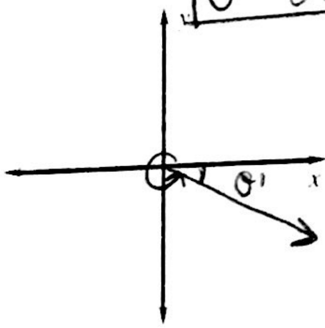
23.) $125°$

$\boxed{\theta' = 55^\circ}$



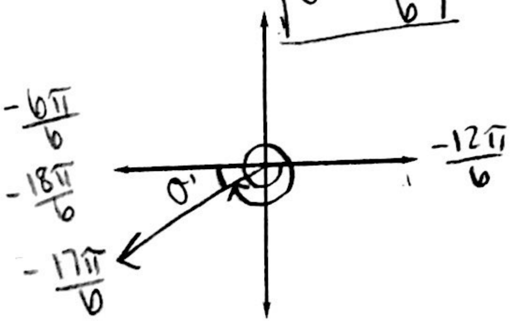
24.) $325°$

$\boxed{\theta' = 35^\circ}$



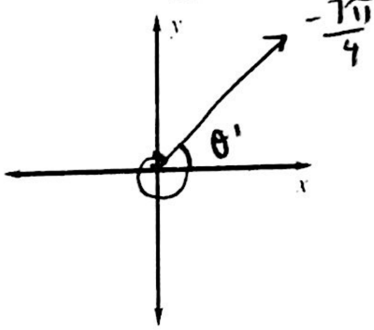
25.) $-\frac{17\pi}{6}$

$\boxed{\theta' = \frac{\pi}{6}}$



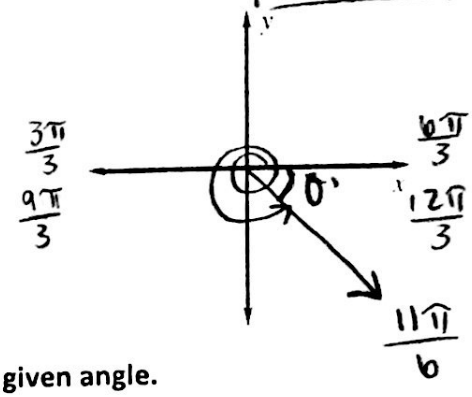
26.) $-\frac{7\pi}{4}$

$\boxed{\theta' = \frac{\pi}{4}}$



27.) $\frac{11\pi}{3}$

$\boxed{\theta' = \frac{\pi}{3}}$



Find one positive angle and one negative angle that are coterminal with the given angle.

28.) $70°$

$70^\circ + 360^\circ = \boxed{430^\circ}$

$70^\circ - 360^\circ = \boxed{-290^\circ}$

29.) $-375°$

$-375^\circ + 2 \cdot 360^\circ = \boxed{345^\circ}$

$-375^\circ + 360^\circ = \boxed{-15^\circ}$

30.) $-\frac{5\pi}{8}$

$-\frac{5\pi}{8} + \frac{16\pi}{8} = \boxed{\frac{11\pi}{8}}$

$-\frac{5\pi}{8} - \frac{16\pi}{8} = \boxed{\frac{-21\pi}{8}}$

31.) $\frac{13\pi}{6}$

$\frac{13\pi}{6} - \frac{12\pi}{6} = \boxed{\frac{\pi}{6}}$

$\frac{13\pi}{6} - 2 \cdot \frac{12\pi}{6} = \boxed{\frac{-11\pi}{6}}$

ert the degree measure to radians or the radian measure to degrees.

32.) $500° \left(\frac{\pi}{180°}\right)$

$\frac{500\pi}{180}$

$\boxed{\frac{25\pi}{9}}$

33.) $-125° \left(\frac{\pi}{180°}\right)$

$\frac{-125\pi}{180}$

$\boxed{\frac{-25\pi}{36}}$

34.) 5π

$5(180°)$

$\boxed{900°}$

35.) $-\frac{\pi}{12}$

$\frac{-180°}{12}$

$\boxed{-15°}$

$S = r\theta \quad A = \frac{1}{2}r^2\theta$

Find the arc length and area of a sector with the given radius *r* and central angle *θ*. Round answers to the nearest hundredth.

36.) $r = 15$ cm, $\theta = 45° \rightarrow 45° \left(\frac{\pi}{180°}\right) = \frac{\pi}{4}$

$S = 15\left(\frac{\pi}{4}\right)$ $\quad A = \frac{1}{2}(15)^2\left(\frac{\pi}{4}\right)$

$\boxed{S \approx 11.78}$ $\quad \boxed{A \approx 88.36}$

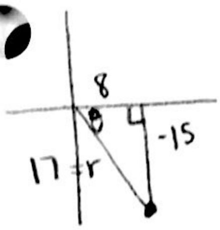
37.) $r = 25$ in., $\theta = 270° \rightarrow 270°\left(\frac{\pi}{180°}\right) = \frac{3\pi}{2}$

$S = 25\left(\frac{3\pi}{2}\right)$ $\quad A = \frac{1}{2}(25)^2\left(\frac{3\pi}{2}\right)$

$\boxed{S \approx 117.81}$ $\quad A \approx \boxed{1472.62}$

Use the given point on the terminal side of an angle *θ* in standard position to evaluate the six trigonometric functions of *θ*.

38.) $(8, -15)$



$(8)^2 + (-15)^2 = r^2$

$r^2 = 289$

$r = 17$

$\sin\theta = \boxed{\frac{-15}{17}}$ $\quad \csc\theta = \boxed{-\frac{17}{15}}$

$\cos\theta = \boxed{\frac{8}{17}}$ $\quad \sec\theta = \boxed{\frac{17}{8}}$

$\tan\theta = \boxed{\frac{-15}{8}}$ $\quad \cot\theta = \boxed{-\frac{8}{15}}$

39.) A projectile is launched with an initial speed of 42 feet per second. It is projected at an angle of 50°. How far does the projectile travel? How much farther does it travel with an initial speed of 84 feet per second?

$d = \frac{v^2}{32} \sin 2\theta$

$d = \frac{(42)^2}{32} \sin(2 \cdot 50°)$

$\boxed{d \approx 54.3 \text{ ft}}$

$d = \frac{(84)^2}{32} \sin(2 \cdot 50°)$

$d \approx 217.2$

$217.2 - 54.3 = \boxed{162.9 \text{ ft further}}$

40.) A baseball player hits a ball projected at an angle of 40°. The height at which the ball is hit is the same as the height of the fence. At what speed must the baseball player hit the ball in order for it to clear a fence that is 385 feet away?

$d = \frac{v^2}{32} \sin 2\theta$

$385 = \frac{v^2}{32} \sin(2 \cdot 40°)$

$385 = \frac{v^2}{32} \sin(80°)$

$32 \cdot 390.94 = \frac{v^2}{32} \cdot 32$

$12510 = v^2$

$v \approx \boxed{111.85 \text{ ft/s}}$