Name: KEY Hour: ____________ Date: ____________

NOTES: Section 7.3 – ~~Graph Exponential Decay Functions~~ Use Functions Involving e

Goals: #1 - I can graph an exponential function with a natural base.

#2 - I can use the natural base in a real life application.

#3 - I can model continuously compounded interest.

Homework: Lesson 7.3 Worksheet

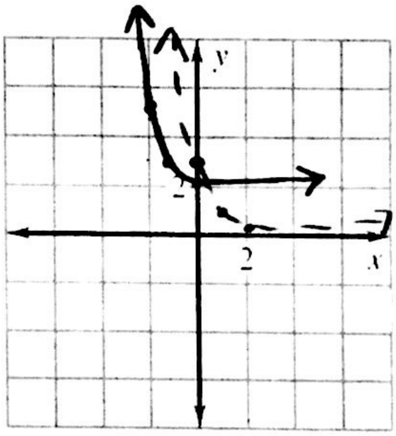
Warm Up:

$3\left(\frac{1}{4}\right)^x$

1. $f(x) = 3\left(\frac{1}{4}\right)^{x+2} + 2$

left 2 up 2

x	y
-1	12
0	3
1	0.8
2	0.2



domain: $(-\infty, \infty)$

range: $(2, \infty)$

2. A new laptop computer costs $1500. The value of the computer decreases by 22% each year.

   a. Write an exponential decay model to represent the situation.

   $y = 1500(1-0.22)^t$

   $\boxed{y = 1500(0.78)^t}$

   b. Estimate the value of the computer after 2 years.

   $y = 1500(0.78)^2$

   $y \approx \boxed{\$912.6}$

Exploration #1: Work with a partner and answer the following questions.

1. Complete the table of vaules by using your calculator.

x	10^1	10^2	10^3	10^4	10^5	10^6
$\left(1+\frac{1}{x}\right)^x$	2.594	2.705	2.717	2.718	2.718	2.718

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Notes:

We have worked with special numbers such as π and i.

Another special number is called the natural base and is denoted by the letter e (the Euler number).

The natural base e is irrational, so we cannot find its exact value. It is defined as:

$$\text{As } n \text{ approaches } +\infty, \left(1+\frac{1}{n}\right)^n \text{ approaches } e \approx 2.718281828$$

Find the e button on your calculator and write the approximation: 2.718281828.

Example #1: Simplify the expression.

1. $e^6 \cdot e^3$

e^{6+3}

$\boxed{e^9}$

2. $\frac{18e^6}{2e^4}$

$9e^{6-4}$

$\boxed{9e^2}$

3. $(4e^{3x})^2$

$4^2 e^{3 \cdot 2x}$

$\boxed{16e^{6x}}$

You practice: Simplify the expression.

1. $2e^{-3} \cdot 6e^5$

$12e^{-3+5}$

$\boxed{12e^2}$

2. $(10e^{-4x})^3$

$10^3 e^{-4 \cdot 3x}$

$1000e^{-12x}$

$\boxed{\frac{1000}{e^{12x}}}$

Example #2: Use a calculator to evaluate the expression.

1. e^{-2}

$\boxed{0.1353}$

2. $e^{0.3}$

$\boxed{1.3499}$

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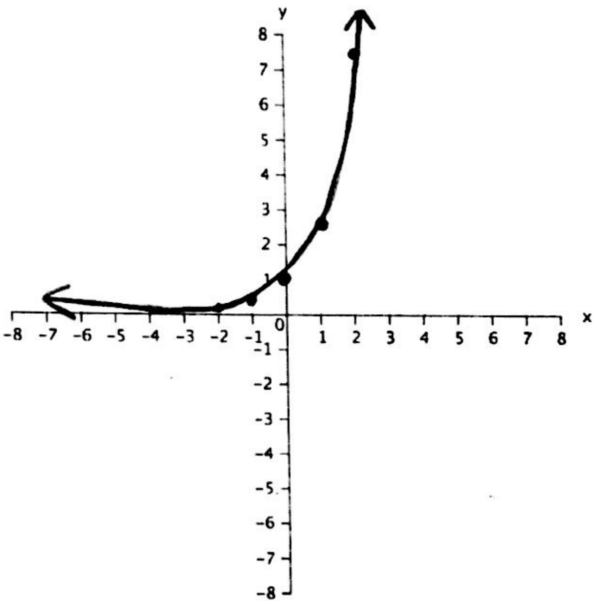
Exploration #2: Work with a partner and answer the following questions.

1. Use your calculator to complete the table.

x	-2	-1	0	1	2
$y = e^x$	0.14	0.37	1	2.72	7.39

2. Graph the function $y = e^x$ using your table. Then state the domain and range.



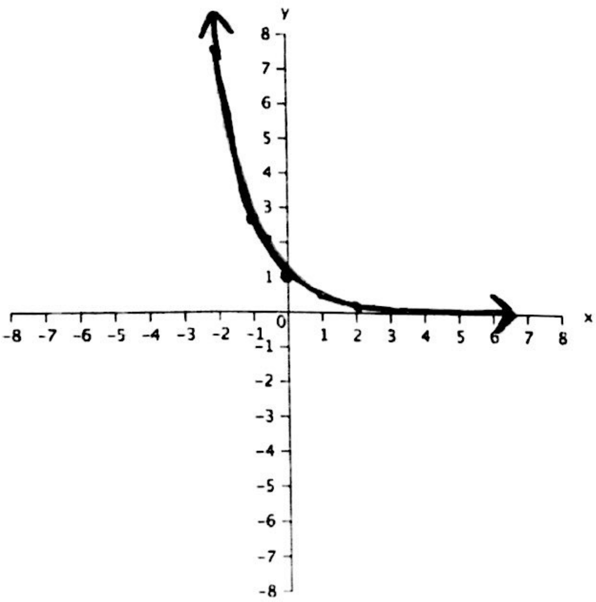
domain: $(-\infty, \infty)$

range: $(0, \infty)$

3. Use your calculator to complete the table.

x	-2	-1	0	1	2
$y = e^{-x}$	7.39	2.72	1	0.37	0.14

4. Graph the function $y = e^{-x}$ using your table. Then state the domain and range.



domain: $(-\infty, \infty)$

range: $(0, \infty)$

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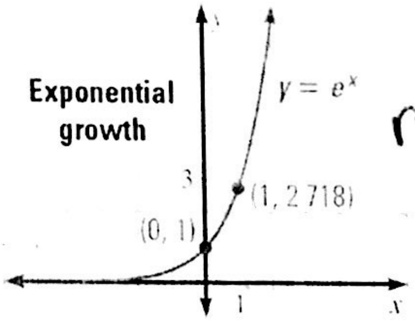
Notes:

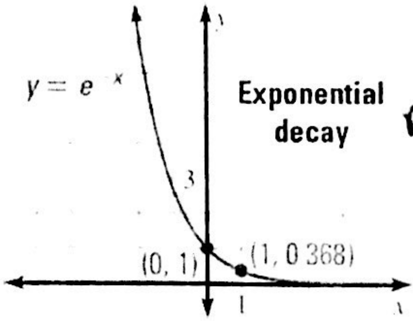
A function of the form $y = ae^{rx}$ is called a natural base exponential function.

- If $r > 0$, the function is an exponential growth function.
- If $r < 0$, the function is an exponential decay function.

The graphs of the basic functions $y = e^{x}$ and $y = e^{-x}$ are shown below.



r=1



r=-1

Example #3: Tell whether the function is an example of *exponential growth* or *exponential decay*.

1. $f(x) = \frac{1}{4}e^{-3x}$ r=-3

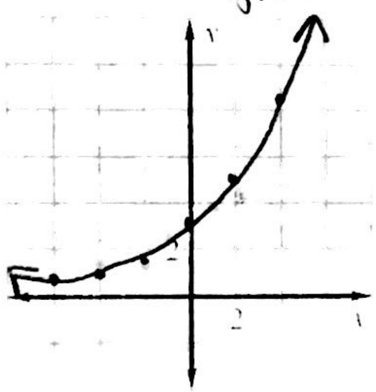
$\boxed{\text{decay}}$

2. $f(x) = 2e^{2x}$ r=2

$\boxed{\text{growth}}$

Example #4: Graph the function. State the domain and range.

1. $y = 3e^{0.25x}$ growth



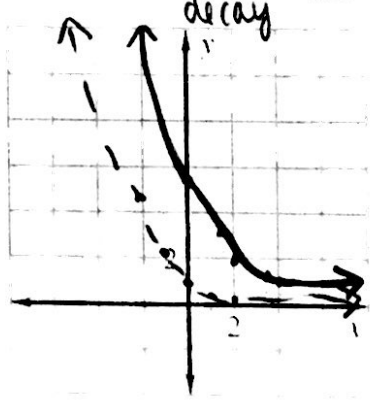
x	y
-6	0.7
-4	1.1
-2	1.8
0	3
2	4.9
4	8.2

domain: $(-\infty, \infty)$

range: $(0, \infty)$

2. $y = e^{-0.75(x-2)} + 1$ right 2 up 1 decay

$e^{-0.75x}$



x	y
-2	4.5
-1	2.1
0	1
1	0.5
2	0.2

domain: $(-\infty, \infty)$

range: $(1, \infty)$

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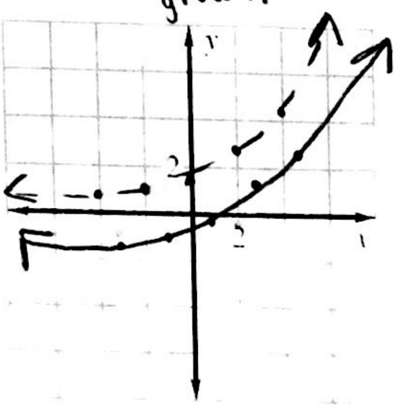
You practice: Graph the function. State the domain and range.

1. $y = 1.5e^{\underline{0.25(x-1)}} \underline{-2}$ down 2

right 1

growth



$y = 1.5e^{0.25x}$

x	y
-4	0.6
-2	0.9
0	1.5
2	2.8
4	4.1

OR $y = 1.5e^{0.25(x-1)} - 2$

x	y
-4	-1.6
-2	-1.3
0	-0.8
2	-0.08
4	1.2
6	3.2

domain: $(-\infty, \infty)$

range: $(2, \infty)$

Notes:

In Section 7.1, we learned about compound interest:

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$

As the number of times interest in compounded increases, it approaches continuously compounded interest which is given by the formula: $A = Pe^{rt}$

principal; t → time (years); r → rate (%)

Example #5: You deposit \$3500 in an account that pays 4% annual interest. What is the balance after 1 year?

1. What is the balance if the interest is compounded monthly?

$A = 3500\left(1 + \frac{0.04}{12}\right)^{12 \cdot 1}$

$A \approx \boxed{\$3642.60}$

2. What is the balance if the interest is compounded continuously?

$A = 3500e^{0.04(1)}$

$A \approx \boxed{\$3642.84}$