Name: KEY Hour: ____________ Date: ____________

NOTES: Section 5.7 – Apply the Fundamental Theorem of Algebra

Goals: #1 - I can identify the number of solutions or zeros in a polynomial.

#2 - I can find all the zeros (real, imaginary, and repeated) in a polynomial.

#3 - I can write a polynomial with given zeros.

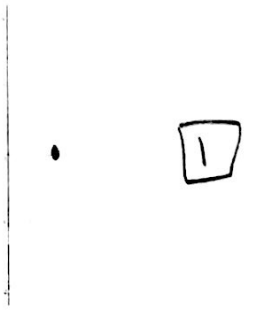
#4 - I can determine the number and type of zeros of a polynomial given the degree and graph.

Homework: Lesson 5.7 Worksheet

Exploration #1: Work with a partner and answer the following questions.

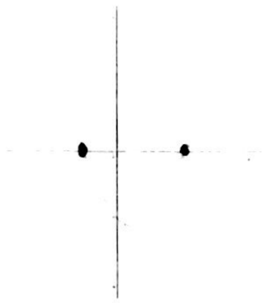
1. How many zeros are in the following graph?

$f(x) = 3x - 2$



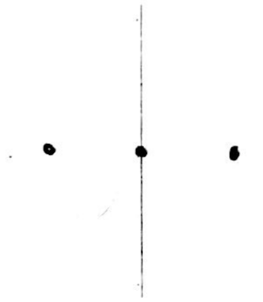
$\boxed{1}$

$f(x) = 2x^2 - x - 1$



$\boxed{2}$

$f(x) = -x^3 + 2x$



$\boxed{3}$

Notes:

- The Fundamental Theorem of Algebra:

  If a polynomial $f(x)$ has a degree n, then the equation $f(x) = 0$ has exactly n solutions, given each repead solution counts as multiple solutions

Example #1: Find the number of solutions or zeros of the following polynomial.

1. $x^3 + 5x^2 + 4x + 20 = 0$ equation

   $\boxed{\text{3 solutions}}$

2. $f(x) = x^4 - 8x^3 + 18x^2 - 27$ function

   $\boxed{\text{4 zeros}}$

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Example #2: Find all zeros of the polynomial function.

1. $f(x) = x^5 - 4x^4 + 4x^3 + 10x^2 - 13x - 14$

$\pm 1, \pm 2, \pm 7, \pm 14$

repeated zero

-1	1	-4	4	10	-13	-14
	↓	-1	5	-9	-1	14
	1	-5	9	1	-14	$\boxed{0}$

$x^4 - 5x^3 + 9x^2 + x - 14$

-1	1	-5	9	1	-14
	↓	-1	6	-15	14
	1	-6	15	-14	$\boxed{0}$

$x^3 - 6x^2 + 15x - 14$

2	1	-6	15	-14
	↓	2	-8	14
	1	-4	7	$\boxed{0}$

$0 = (x+1)^2(x-2)(x^2-4x+7)$

not factorable!

$(x+1)^2 = 0$ $\boxed{x=-1}$

$x - 2 = 0$ $\boxed{x=2}$

$x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(7)}}{2(1)}$

$x = \frac{4 \pm \sqrt{-12}}{2}$

$x = \frac{4 \pm 2i\sqrt{3}}{2}$

$\boxed{x = 2 \pm i\sqrt{3}}$

You practice: Find all zeros of the polynomial function.

1. $f(x) = x^5 - 2x^4 + 8x^2 - 13x + 6$

$\pm 1, \pm 2, \pm 3, \pm 6$

1	1	-2	0	8	-13	6
	↓	1	-1	-1	7	-6
	1	-1	-1	7	-6	$\boxed{0}$

$x^4 - x^3 - x^2 + 7x - 6$

-2	1	-1	-1	7	-6
	↓	-2	6	-10	6
	1	-3	5	-3	$\boxed{0}$

$x^3 - 3x^2 + 5x - 3$

1	1	-3	5	-3
	↓	1	-2	3
	1	-2	3	$\boxed{0}$

$0 = (x-1)^2(x+2)(x^2-2x+3)$

not factorable

$(x-1)^2 = 0$ $\boxed{x=1}$

$x+2=0$ $\boxed{x=-2}$

$x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(3)}}{2(1)}$

$x = \frac{2 \pm \sqrt{-8}}{2}$

$x = \frac{2 \pm 2i\sqrt{2}}{2}$

$\boxed{x = 1 \pm i\sqrt{2}}$

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Notes:

- Conjugates Theorem:
  - If a polynomial $f(x)$ has $a+bi$ as an imaginary zero, then $a-bi$ is also a zero of f. (complex conjugate)
  - If a polynomial $f(x)$ has $a+\sqrt{b}$ as an imaginary zero, then $a-\sqrt{b}$ is also a zero of f. (irrational conjugate)

Example #3: Write a polynomial function f of at least degree that has rational coefficients, a leading coefficient of 1, and 3 and $2+\sqrt{5}$ as zeros.

$(x-3) \quad (x-(2+\sqrt{5})) \quad (x-(2-\sqrt{5}))$

$$f(x) = (x-3)((x-2)+\sqrt{5})((x-2)-\sqrt{5})$$

$$f(x) = (x-3)((x-2)^2-5)$$ FOIL

$$f(x) = (x-3)(x^2-4x+4-5)$$

$$f(x) = (x-3)(x^2-4x-1)$$

$$f(x) = x^3-4x^2-x-3x^2+12x+3$$

$$\boxed{f(x) = x^3-7x^2+11x+3}$$

You practice: Write a polynomial function f of at least degree that has rational coefficients, a leading coefficient of 1, and 2, $2i$, and $4-\sqrt{6}$ as zeros.

$(x-2)\ (x+2i)(x-2i)(x-(4-\sqrt{6}))(x-(4+\sqrt{6}))$

$$f(x) = (x-2)(x+2i)(x-2i)((x-4)-\sqrt{6})((x-4)+\sqrt{6})$$

$$f(x) = (x-2)(x^2-4i^2)((x-4)^2-6)$$

$$f(x) = (x-2)(x^2+4)(x^2-8x+16-6)$$

$$f(x) = (x^3+4x-2x^2-8)(x^2-8x+10)$$

$$f(x) = (x^2-8x+10)(x^3-2x^2+4x-8)$$

$$f(x) = x^5-2x^4+4x^3-8x^2-8x^4+16x^3-32x^2+64x+10x^3-20x^2+40x-80$$

$$\boxed{f(x) = x^5-10x^4+30x^3-60x^2+104x-80}$$

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