

NOTES: Section 4.3 – Solve $x^2 + bx + c = 0$ by Factoring

Goals: #1 - I can factor a quadratic in the form $ax^2 + bx + c$ when $a = 1$

#2 - I can factor a difference of two squares.



#3 - I can factor a perfect square trinomial.

#4 - I can use the zero product property to solve $ax^2 + bx + c = 0$ by factoring when $a = 1$

Homework: Lesson 4.3 Worksheet

Warm Up: Graph each function on the same coordinate plane. Identify the graph's axis of symmetry, vertex, y -intercept, whether the graph opens up or down, and its maximum/minimum value.

1. $f(x) = -2(x + 2)^2 + 6$

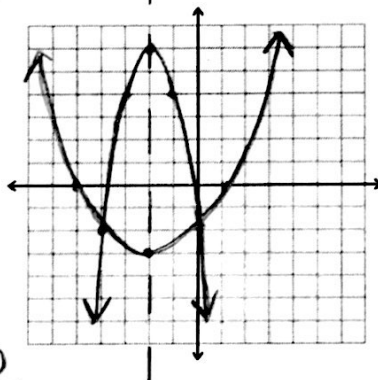
AOS: $x = -2$

vertex: $(-2, 6)$

y -int: $(0, -2)$

opens: down ↴

max./min. value: $y = 6$



x	-4	-3	-2	-1	0
y	-2	4	6	4	-2

work: $y = -2(0+2)^2 + 6$
 $= -2(4) + 6 = -2$

Exploration #1: Work with a partner. Find the product.

1. $(m - 8)(m - 9)$

$m^2 - 9m - 8m + 72$
 $m^2 - 17m + 72$

2. $g(x) = \frac{1}{3}(x - 1)(x + 5)$ x -int: $(1, 0)$
 $(-5, 0)$

AOS: $x = -2$

vertex: $(-2, -3)$

y -int: $(0, -1\frac{2}{3})$

opens: up ↴

max./min. value: $y = -3$

$y = \frac{1}{3}(0-1)(0+5)$
 $= \frac{1}{3}(-1)(5)$
 $= -1\frac{2}{3}$

x					
y					

work: $x = \frac{p+q}{2} = \frac{-5+1}{2} = \frac{-4}{2} = -2$
 $y = \frac{1}{3}(-2-1)(-2+5)$
 $= \frac{1}{3}(-3)(3) = -3$

2. $(y + 20)(y - 20)$

$y^2 - 20y + 20y - 400$
 $y^2 - 400$

CHALLENGE: Can you go backwards? Break $x^2 - 9x + 20$ into factors.

$(x - 5)(x - 4)$
 $x^2 - 5x - 4x + 20$
 $x^2 - 9x + 20$

Notes:

A monomial is an expression that is either a number, a variable, or the product of a number and one or more variables. (1 term)

Examples: 7, x, 5x, 4xy²

A binomial is the sum of two monomials. (2 terms)

Examples: 2x + 7, x - y, 3xy² + 8nb⁴

A trinomial is the sum of three monomials. (3 terms)

Examples: x + y + z, x² + 6x + 5, 5x² + 4x + 7x + 8, 5x² + 7y + 1
5x² + 11x + 8

Example #1: Factor the expression.

1. $x^2 - 9x + 20$

Factors of 20:

1 + 20 = 21

2 + 10 = 12

4 + 5 = 9

-4 + -5 = -9

2. $x^2 + 3x - 12$

Factors of -12:

1 + -12 = -11

3 + -4 = -1

2 + -6 = -4

-1 + 12 = 11

-3 + 4 = 1

-2 + 6 = 4

not factorable

* looking for 2 numbers that add to -9 and multiply to 20

(x - 4)(x - 5)

check:

$x^2 - 5x - 4x + 20$

$x^2 - 9x + 20$ ✓

3. $x^2 - 3x - 18$

Factors of -18:

1 + -18 = -17

2 + -9 = -7

3 + -6 = -3

4. $r^2 + 2r - 63$

Factors of -63:

1 + -63 = -62

-3 + 21 = 18

-9 + 7 = -2

9 + -7 = 2

(r - 7)(r + 9)

check:

$x^2 + 3x - 6x - 18$

$x^2 - 3x - 18$ ✓

check:

$(r - 7)(r + 9)$

$r^2 + 9r - 7r - 63$

$r^2 + 2r - 63$ ✓

Notes:

There are special factoring patterns we can look for!

• Difference of two squares: $a^2 - b^2 = (a+b)(a-b)$
 Examples: $x^2 - 16 = (x-4)(x+4)$ $x^2 - 64 = (x+8)(x-8)$

• Perfect Square Trinomial: $a^2 + 2ab + b^2 = (a+b)^2$
 $a^2 - 2ab + b^2 = (a-b)^2$
 Examples: $x^2 + 6x + 9 = (x+3)^2$ $x^2 - 4x + 4 = (x-2)^2$

Example #2: Factor the expression.

1. $x^2 - 49$
 \uparrow
 7^2
 $(x+7)(x-7)$

2. $d^2 + 12d + 36$
 \uparrow \uparrow
 $2(b)$ b^2
 $(d+b)^2$

Factors 36:
 $2 + 18 = 20$
 $3 + 12 = 15$
 $b + b = 12$
 $(d+b)(d+b)$
 $= (d+b)^2$

check:
 $(x+7)(x-7)$
 $= x^2 - 7x + 7x - 49$
 $= x^2 - 49$

3. $q^2 - 9$
 \uparrow
 3^2
 $(q-3)(q+3)$

4. $y^2 + 16y + 64$
 \uparrow \uparrow
 $2(b)$ b^2
 $(y+8)^2$

Notes:

We can use factoring to solve certain quadratic equations.

We set the quadratic equation equal to 0 and use the zero product property

- **Zero Product Property:** if $AB=0$, then $A=0$ or $B=0$

The solutions of a quadratic equation are called the roots of the equation.

Example #3: Solve the equation.

1. $x^2 + 2x - 35 = 0$

$(x+7)(x-5) = 0$

ZPP: $x+7=0$ OR $x-5=0$
 $-7 \quad -7$ $+5 \quad +5$
 $x = -7$ $x = 5$

2. $u^2 = -9u$
 $+9u \quad +9u$

$u^2 + 9u = 0$

$u(u+9) = 0$

$u = 0$ OR $u+9=0$
 $-9 \quad -9$

$u = -9$

Example #4: Find the roots of the equation. (Solve)

1. $r^2 + 2r = 80$
 $-80 \quad -80$

$r^2 + 2r - 80 = 0$

$(r+10)(r-8) = 0$

$r+10=0$ $r-8=0$
 $-10 \quad -10$ $+8 \quad +8$
 $r = -10$ $r = 8$

2. $a^2 - 49 = 0$

* difference of squares!

$(a+7)(a-7) = 0$

$a+7=0$
 $-7 \quad -7$

$a-7=0$
 $+7 \quad +7$

$a = -7$

$a = 7$

Exploration #1: Work with a partner and answer the following questions.

1. Rewrite the quadratic function in intercept form: $y = x^2 - x - 12$

$y = x^2 - x - 12$ $\rightarrow y = a(x-p)(x-q)$
 $y = (x-4)(x+3)$

2. Graph the function you found in #1.

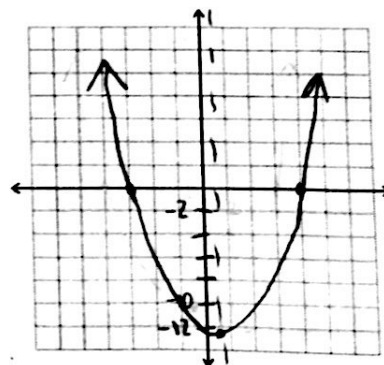
a. x-int(s): $(4, 0)$ $(-3, 0)$

$x = \frac{4+(-3)}{2} = \frac{1}{2}$

$y = (\frac{1}{2})^2 - (\frac{1}{2}) - 12 = -12.25$

b. What is the y-value of the x-intercepts?

0



Notes:

Recall the intercept form of a quadratic function: $y = a(x-p)(x-q)$

Because quadratic function's values are 0 when $X=p$ and $X=q$, these are also called ZEROS of the function.

Example #5: Find the zeros of the function by rewriting the function in intercept form.

1. $y = x^2 + 12x + 36$

$$y = (x + 6)(x + 6)$$

$$y = (x + 6)^2$$

$$0 = (x + 6)^2$$

$$0 = x + 6$$

$$x = -6$$

x-int: $(0, -6)$

-6

2. $y = x^2 - 7x - 30$

$$y = (x - 10)(x + 3)$$

$$x\text{-int: } (0, 10) \quad (0, -3)$$

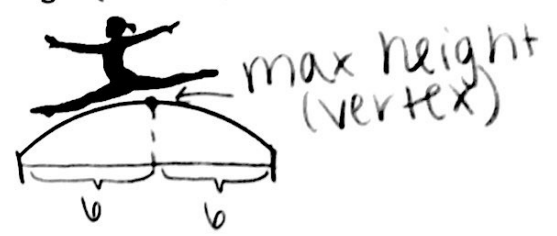
-3 : 10

Example #6: The function $y = -1.17(x - 6)^2 + 42$ models the leap of a gymnast where x is the horizontal distance (in inches) and y is the corresponding height (in inches). What is the gymnast's maximum height? How far does she leap?

vertex: $(6, 42)$

↑ ↑

distance height



$y =$ 42 inches max height

$6 + 6 =$ 12 inches how far