Name: KEY Hour: ____________ Date: ____________

NOTES: Section 13.3 – Evaluate Trigonometric Functions of Any Angle

Goals: #1 - I can evaluate the 6 trig functions for a quadrantal function without using a calculator.

#2 - I can find the reference angle for any given angle, in both degrees and radians.

#3 - I can evaluate trig functions for special angles (multiples of 30° and 45°) in quadrants 1, 2, 3, and 4 without using a calculator.

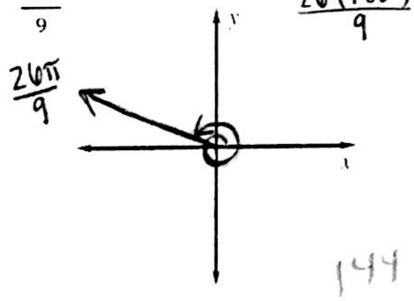
#4 - I can apply the formula for horizontal distance of a projectile launched in terms of initial velocity and launch angle.

Homework: Lesson 13.3 Worksheet

Warm Up:

1. Draw an angle with the given measure in standard position.

a. $\frac{26\pi}{9}$ $\quad \frac{26(180^\circ)}{9} = 520^\circ$

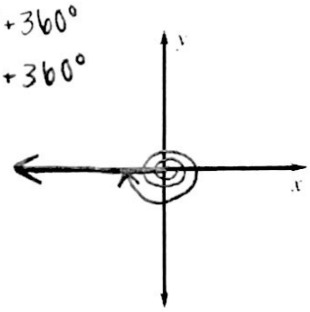


144

b. $-900^\circ + 360^\circ$

$= -540^\circ + 360^\circ$

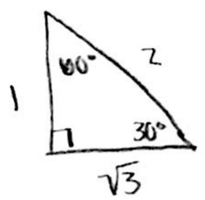
$= -180^\circ$



2. Evaluate the trigonometric function. When possible, give an exact answer. When using a calculator, round answers to the nearest <u>hundredth</u>.

a. $\tan\frac{\pi}{6}$ $\rightarrow \frac{180^\circ}{6}$

$\rightarrow 30^\circ$



$\tan 30^\circ$

$= \frac{1}{\sqrt{3}}$

$= \boxed{\frac{\sqrt{3}}{3}}$

b. $\csc\frac{4\pi}{15}$ $\rightarrow \frac{4(180^\circ)}{15}$

$\rightarrow 48^\circ$

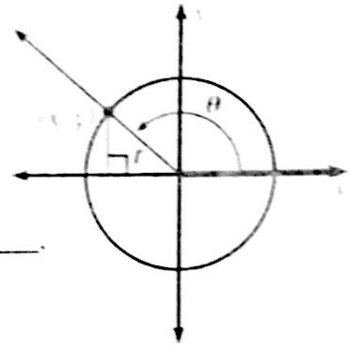
$\csc 48^\circ$

$\frac{1}{\sin 48^\circ}$

$\approx \boxed{1.35}$

Name:______________________ Hour: ____________ Date: ____________

Notes:



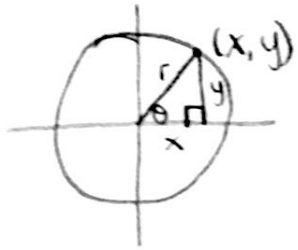
We can evaluate trigonometric functions of ANY angle.

Let θ be an angle in standard position, and let (x, y) be the point where the terminal side of θ intersects the circle $x^2 + y^2 = r^2$.

$\sin\theta = \frac{y}{r}$ $\csc\theta = \frac{r}{y}$

$\cos\theta = \frac{x}{r}$ $\sec\theta = \frac{r}{x}$

$\tan\theta = \frac{y}{x}$ $\cot\theta = \frac{x}{y}$

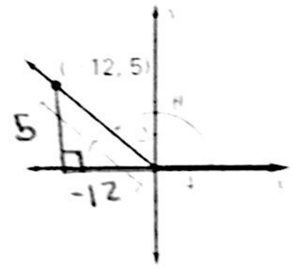


Example #1: Let $(-12, 5)$ be a point on the terminal side of an angle θ in standard position. Evaluate the six trigonometric functions of θ.

$\sin\theta = \frac{5}{13}$ $\csc\theta = \frac{13}{5}$

$\cos\theta = \frac{-12}{13}$ $\sec\theta = \frac{13}{-12}$

$\tan\theta = \frac{5}{-12}$ $\cot\theta = \frac{-12}{5}$



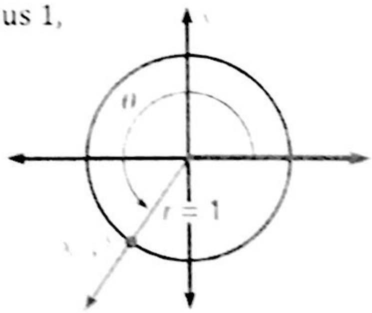
$(5)^2 + (-12)^2 = r^2$

$169 = r^2$

$r = 13$

Notes:

The circle $x^2 + y^2 = 1$, which has center $(0, 0)$ and radius 1, is called the unit circle.



$\sin\theta = \frac{y}{r} = \frac{y}{1} = y$ $\cos\theta = \frac{x}{r} = \frac{x}{1} = x$

$\tan\theta = \frac{y}{x}$

A quandrantal is an angle in standard position whose Terminal side lies on an axis. The meausre is always a multiple of $90°$ or $\frac{\pi}{2}$.

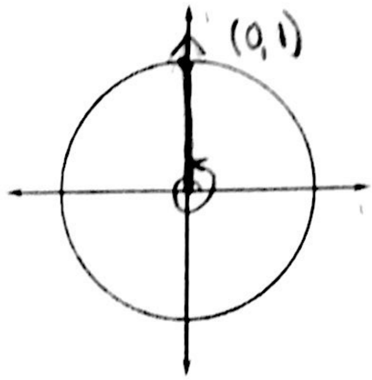
Name:____________________ Hour: __________ Date: __________

Example #2: Use the unit circle to evaluate the six trigonometric functions of $\theta = 450°$

$\sin\theta = y = 1$ $\quad$ $\sec\theta = 1$

$\cos\theta = x = 0$ $\quad$ $\sec\theta = \frac{1}{0}$ undefined

$\tan\theta = \frac{y}{x} = \frac{1}{0}$ undefined $\quad$ $\cot\theta = \frac{0}{1} = 0$

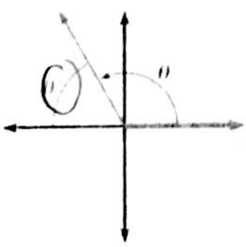


Notes:

How can we find a trig function of ANY angle? We use reference angles.

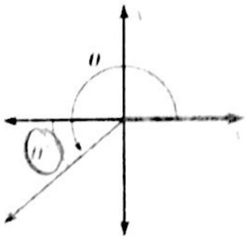
The reference angle for θ is the acute angle formed by the terminal side of θ and the x-axis.

Quadrant II



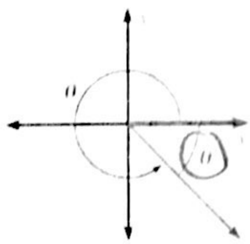
Degrees: $\theta' = 180° - \theta$
Radians: $\theta' = \pi - \theta$

Quadrant III



Degrees: $\theta' = \theta - 180°$
Radians: $\theta' = \theta - \pi$

Quadrant IV

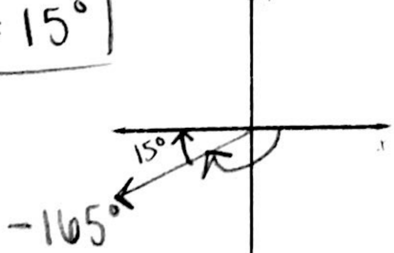


Degrees: $\theta' = 360° - \theta$
Radians: $\theta' = 2\pi - \theta$

Example #3: Sketch the angle. Then find its reference angle. Answer in the unit of the given angle.

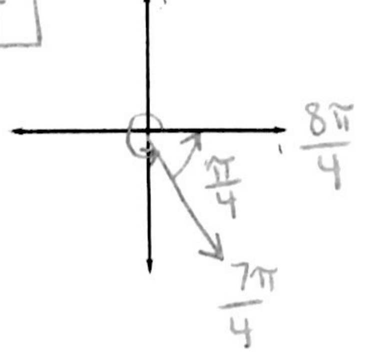
1. $\theta = -165°$

$\boxed{\theta' = 15°}$



2. $\theta = \frac{7\pi}{4}$ $\quad$ $\frac{7(180°)}{4} = 315°$

$\boxed{\theta' = \frac{\pi}{4}}$



Name:______________________ Hour: __________ Date: __________

You practice:

1. Use the unit circle to evaluate the six trigonometric functions of $\theta = 4\pi$

$\sin\theta = 0$

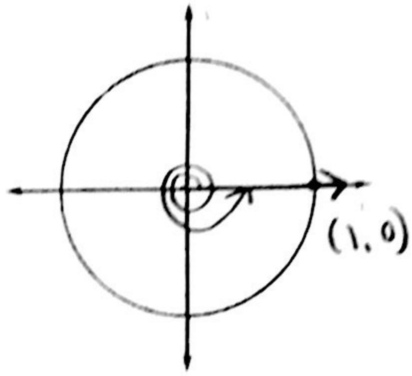
$\cos\theta = 1$

$\tan\theta = \frac{0}{1} = 0$

$\csc\theta = \cancel{0}$ undefined

$\sec\theta = 1$

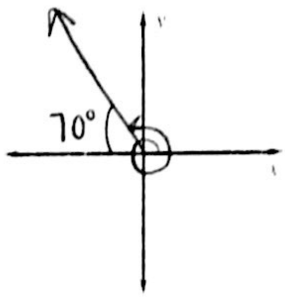
$\cot\theta = \cancel{0}$ undefined



2. Sketch the angle. Then find its reference angle. Answer in the unit of the given angle.

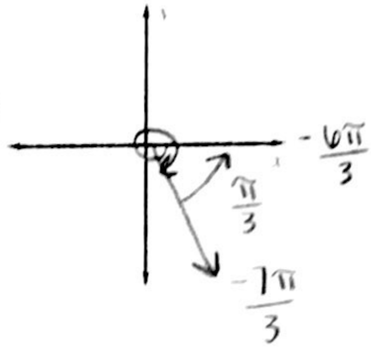
a. $\theta = 470°$

$\boxed{\theta' = 70°}$



b. $\theta = -\frac{7\pi}{3}$ $-420°$

$\boxed{\theta' = \frac{\pi}{3}}$



Notes:

Finally we can evaluate <u>ANY</u> trig function for <u>ANY</u> θ angle

STEP 1: Find reference angle (θ')

STEP 2: Evaluate the trig function for θ'

STEP 3: Determine the sign of the trig function from what quadrant θ lies in

UNIT CIRCLE

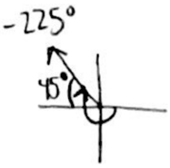
Signs of Function Values

Quadrant II	Quadrant I
$\sin\theta, \csc\theta$ +	$\sin\theta, \csc\theta$ +
$\cos\theta, \sec\theta$ −	$\cos\theta, \sec\theta$ +
$\tan\theta, \cot\theta$ −	$\tan\theta, \cot\theta$ +
Quadrant III	**Quadrant IV**
$\sin\theta, \csc\theta$ −	$\sin\theta, \csc\theta$ −
$\cos\theta, \sec\theta$ −	$\cos\theta, \sec\theta$ +
$\tan\theta, \cot\theta$ +	$\tan\theta, \cot\theta$ −

Name:____________________ Hour: __________ Date: __________

Example #4: Evaluate the following trig functions.

$\frac{10\pi}{3} - \frac{6\pi}{3} = \frac{4\pi}{3}$



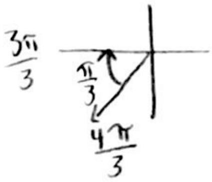
1. $\cos(-225°)$

$\cos 45°$

$= \frac{\sqrt{2}}{2}$

$\cos(-225°)$

$= \boxed{-\frac{\sqrt{2}}{2}}$



2. $\cot\frac{10\pi}{3}$

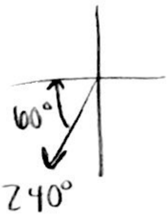
$\cot\frac{\pi}{3}$

$= \frac{1}{\tan\frac{\pi}{3}} = \frac{\frac{\sqrt{3}}{2}}{\frac{1}{2}} = \frac{\sqrt{3}}{2}\cdot\frac{2}{1} = \sqrt{3}$

$= \frac{1}{\sqrt{3}}$

$\boxed{\frac{\sqrt{3}}{3}}$

You practice: Evaluate the following trig functions.



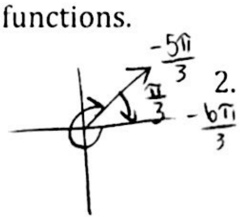
1. $\tan(240°)$

$\tan 60°$

$= \frac{\frac{\sqrt{3}}{2}}{\frac{1}{2}}$

$= \frac{\sqrt{3}}{2}\cdot\frac{2}{1}$

$= \boxed{\sqrt{3}}$



2. $\sec\frac{-5\pi}{3}$

$\sec\frac{\pi}{3}$

$= \frac{1}{\cos\frac{\pi}{3}}$

$= \frac{1}{\frac{1}{2}}$

$= \boxed{2}$

Notes:

The horizontal distance d (in feet) traveled by a projectile launced at an angle θ and with an initial speed v (in feet per second) is given by: $d = \frac{v^2}{32}\sin 2\theta$

Example #5: You kick a soccer ball at an intial speed of 46 feet per second, projected an an angle of 30°. How far will the ball travel horizontally before hitting the ground?

$d = \frac{(46)^2}{32}\sin(2\cdot 30°)$

$= \frac{2116}{32}\sin 60°$

$= \frac{2116}{32}\cdot\frac{\sqrt{3}}{2}$

$= \frac{2116\sqrt{3}}{64} \approx \boxed{57.27\text{ ft}}$