Name: LEY Hour: ____________ Date: ____________

NOTES: Section 5.6 – Find Rational Zeros

Goals: #1 - I can find the possible rational zeros of a polynomial. 😎 😬 😢

#2 - I can find all the real zeros of a polynomial.

Homework: Lesson 5.6 Worksheet

Warm Up:

1. Divide: $(6x^4 - x^3 - x^2 + 11x - 18) \div (2x^2 + x - 3)$

$3x^2 - 2x + 5$

$2x^2+x-3 \overline{)\, 6x^4 - x^3 - x^2 + 11x - 18}$

$-\ \cancel{6x^4} + 3x^3 - 9x^2$ ↓

$-\cancel{4x^3} + 8x^2 + 11x$

$-\ -\cancel{4x^3} - 2x^2 + 6x$ ↓

$\cancel{10x^2} + \cancel{5x} - 18$

$-\ 10x^2 + 5x - 15$

-3 R

$$\boxed{3x^2 - 2x + 5 + \frac{-3}{2x^2+x-3}}$$

2. One zero of $f(x) = x^3 - x^2 - 17x - 15$ is $x = -1$. What are the other two zeros of the function?

↘ $(x+1)$

$$\begin{array}{r|rrrr} -1 & 1 & -1 & -17 & -15 \\ & \downarrow & -1 & 2 & 15 \\ \hline & 1 & -2 & -15 & \underline{\lvert 0} \end{array}$$

$0 = (x+1)(x^2 - 2x - 15)$

$0 = (x+1)(x-5)(x+3)$

$x+1=0$ $x-5=0$ $x+3=0$

$\boxed{x=-1}$ $\boxed{x=5}$ $\boxed{x=-3}$

Notes:

- Rational Zero Theorem:

If a polynomial $f(x)$ has integer coefficients, then every
↘ +: - whole #s

rational zero of f has the following form:

$$\frac{p}{q} = \frac{\text{factors of constant term}}{\text{factors of leading coefficient}}$$

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Example #1: List the possible rational zeros of f using the rational zero theroem.

1. $f(x) = \underset{q}{\underline{x}}^3 + 2x^2 - 11x + \underset{p}{\underline{12}}$

factors of 12: ±1, ±2, ±3, ±4, ±6, ±12

factors of 1: ±1

possible rational zeros $\left(\frac{p}{q}\right)$: $\pm\frac{1}{1}, \pm\frac{2}{1}, \pm\frac{3}{1}, \pm\frac{4}{1}, \pm\frac{6}{1}, \pm\frac{12}{1}$

: $\boxed{\pm1, \pm2, \pm3, \pm4, \pm6, \pm12}$

2. $f(x) = \underset{q}{\underline{4}}x^4 - x^3 - 3x^2 + 9x - \underset{p}{\underline{10}}$

-10: ±1, ±2, ±5, ±10

4: ±1, ±2, ±4

$\frac{p}{q}$: $\pm\frac{1}{1}, \pm\frac{2}{1}, \pm\frac{5}{1}, \pm\frac{10}{1},$

$\pm\frac{1}{2}, \pm\frac{2}{2}, \pm\frac{5}{2}, \pm\frac{10}{2},$

$\pm\frac{1}{4}, \pm\frac{2}{4}, \pm\frac{5}{4}, \pm\frac{10}{4}$

: $\boxed{\pm1, \pm2, \pm5, \pm10, \pm\frac{1}{2}, \pm\frac{5}{2}, \pm\frac{1}{4}, \pm\frac{5}{4}}$

Example #2: Find all real zeros of $f(x) = \underset{q}{\underline{x}}^3 - 8x^2 + 11x + \underset{p}{\underline{20}}$

20: ±1, ±2, ±4, ±5, ±10, ±20

1: ±1

$\frac{p}{q}$: $\pm\frac{1}{1}, \pm\frac{2}{1}, \pm\frac{4}{1}, \pm\frac{5}{1}, \pm\frac{10}{1}, \pm\frac{20}{1}$

: ±1, ±2, ±4, ±5, ±10, ±20

$0 = (x+1)(x^2-9x+20)$

$0 = (x+1)(x-5)(x-4)$

$\boxed{x=-1}$ $\boxed{x=5}$ $\boxed{x=4}$

Test x=1:

$$\begin{array}{r|rrrr} 1 & 1 & -8 & 11 & 20 \\ & \downarrow & 1 & -7 & 4 \\ \hline & 1 & -7 & 4 & \boxed{24} \end{array} \rightarrow \text{not a zero!}$$

Test x=-1:

$$\begin{array}{r|rrrr} -1 & 1 & -8 & 11 & 20 \\ & \downarrow & -1 & 9 & -20 \\ \hline & 1 & -9 & 20 & \boxed{0} \end{array} \rightarrow \text{a zero!}$$

You practice: Find all real zeros of $f(x) = \underset{q}{\underline{x}}^3 - 4x^2 - 15x + \underset{p}{\underline{18}}$

18: ±1, ±2, ±3, ±6, ±9, ±18

1: ±1

$\frac{p}{q}$: $\pm\frac{1}{1}, \pm\frac{2}{1}, \pm\frac{3}{1}, \pm\frac{6}{1}, \pm\frac{9}{1}, \pm\frac{18}{1}$

: ±1, ±2, ±3, ±6, ±9, ±18

$0 = (x-1)(x^2-3x-18)$

$0 = (x-1)(x-6)(x+3)$

$\boxed{x=1}$ $\boxed{x=6}$ $\boxed{x=-3}$

Test x=1:

$$\begin{array}{r|rrrr} 1 & 1 & -4 & -15 & 18 \\ & \downarrow & 1 & -3 & -18 \\ \hline & 1 & -3 & -18 & \boxed{0} \end{array}$$

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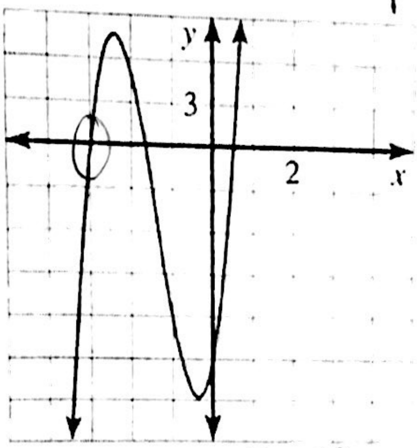
Example #3: Use the graph to help find all real zeros of the function.

1. $f(x) = \underset{q}{\underline{6}}x^3 + 25x^2 + 16x \underset{p}{\underline{-15}}$



p: ±1, ±3, ±5, ±15

q: ±1, ±2, ±3, ±6

$\frac{p}{q}$: $\pm\frac{1}{1}, \pm\frac{3}{1}, \pm\frac{5}{1}, \pm\frac{15}{1}, \pm\frac{1}{2}, \pm\frac{3}{2}, \pm\frac{5}{2}, \pm\frac{15}{2},$
$\pm\frac{1}{3}, \pm\frac{3}{3}, \pm\frac{5}{3}, \pm\frac{15}{3}, \pm\frac{1}{6}, \pm\frac{3}{6}, \pm\frac{5}{6}, \pm\frac{15}{6}$

: $\pm1, \pm3, \pm5, \pm15, \pm\frac{1}{2}, \pm\frac{3}{2}, \pm\frac{5}{2}, \pm\frac{15}{2},$
$\pm\frac{1}{3}, \pm\frac{5}{3}, \pm\frac{1}{6}, \pm\frac{5}{6}$

$$\begin{array}{r|rrrr} -3 & 6 & 25 & 16 & -15 \\ & \downarrow & -18 & -21 & 15 \\ \hline & 6 & 7 & -5 & \boxed{0} \end{array}$$

$6 \cdot -5 = -30$
$7 = 10 , -3$

$0 = (x+3)(6x^2 + 7x - 5)$

$0 = (x+3)(\underbrace{6x^2 + 10x}\underbrace{-3x - 5})$

$0 = (x+3)(2x(3x+5) - 1(3x+5))$

$0 = (x+3)(3x+5)(2x-1)$

$\boxed{x = -3}$ $\boxed{x = -5/3}$ $\boxed{x = 1/2}$

Example #2: Find all real zeros of $f(x) = \underset{q}{\underline{3}}x^4 - 6x^3 - 32x^2 + 35x \underset{p}{\underline{-12}}$

12: ±1, ±2, ±3, ±4, ±6, ±12

3: ±1, ±3

$\frac{p}{q}$: $\pm\frac{1}{1}, \pm\frac{2}{1}, \pm\frac{3}{1}, \pm\frac{4}{1}, \pm\frac{6}{1}, \pm\frac{12}{1}$
$\pm\frac{1}{3}, \pm\frac{2}{3} \pm\frac{3}{3}, \pm\frac{4}{3}, \pm\frac{6}{3}, \pm\frac{12}{3}$

: ±1, ±2, ±3, ±4, ±6, ±12,
$\pm\frac{1}{3}, \pm\frac{2}{3}, \pm\frac{4}{3}$

$\boxed{x = -3, 4}$

$$\begin{array}{r|rrrrr} 1 & 3 & -6 & -32 & 35 & -12 \\ & \downarrow & 3 & -3 & -35 & 0 \\ \hline & 3 & -3 & -35 & 0 & \cancel{\boxed{-12}} \end{array}$$

$$\begin{array}{r|rrrrr} -3 & 3 & -6 & -32 & 35 & -12 \\ & \downarrow & -9 & 45 & -39 & 12 \\ \hline & 3 & -15 & 13 & -4 & \boxed{0} \end{array}$$

$\underset{q}{3} \quad -15 \quad 13 \quad \underset{p}{-4}$

$\frac{\pm1, \pm2, \pm4}{\pm1, \pm3} = \pm1, \pm2, \pm4, \pm\frac{1}{3}, \pm\frac{2}{3}, \pm\frac{1}{2}$

$$\begin{array}{r|rrrr} 4 & 3 & -15 & 13 & -4 \\ & \downarrow & 12 & -12 & 4 \\ \hline & 3 & -3 & 1 & \boxed{0} \end{array}$$

$0 = 3x^2 - 3x + 1$

$x = \frac{3 \pm \sqrt{(-3)^2 - 4(3)(1)}}{2(3)}$

$\cancel{x = \frac{3 \pm \sqrt{-3}}{6}}$

You practice: Find all real zeros of $f(x) = \underset{q}{\underline{2}}x^3 + 5x^2 - 11x \underset{p}{\underline{-14}}$

14: ±1, ±2, ±7, ±14

2: ±1, ±2

$\frac{p}{q}$: $\pm\frac{1}{1}, \pm\frac{2}{1}, \pm\frac{7}{1}, \pm\frac{14}{1}$
$\pm\frac{1}{2}, \pm\frac{2}{2}, \pm\frac{7}{2}, \pm\frac{14}{2}$

: ±1, ±2, ±7, ±14,
$\pm\frac{1}{2}, \pm\frac{7}{2}$

$$\begin{array}{r|rrrr} -1 & 2 & 5 & -11 & -14 \\ & \downarrow & -2 & -3 & 14 \\ \hline & 2 & 3 & -14 & \boxed{0} \end{array}$$

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$7 + -4 = 3$

$0 = (x+1)(2x^2 + 3x - 14)$

$0 = (x+1)(2x^2 + 7x - 4x - 14)$

$0 = (x+1)(x(2x+7) - 2(2x+7))$

$0 = (x+1)(2x+7)(x-2)$

$\boxed{x = -1}$ $\boxed{x = \frac{-7}{2}}$ $\boxed{x = 2}$