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# NOTES: Section 5.2 – Evaluate and Graph Polynomial Functions

Goals: #1 - I can determine whether a function is polynomial and if so, state it's degree, type, and leading coefficient.

#2 - I can use direct substitution to evaluate a polynomial for the given value of x.

#3 - I can use synthetic substitution to evaluate a polynomial for the given value of x.

#4 - I can describe the end behavior (general shape) of the graph of a polynomial by looking at its equation.

*Homework: Lesson 5.2 Worksheet*

**Warm Up:** Simplify the expression.

1. $\frac{-14x^{-3}y^5}{35xy^3}$

$-\frac{2x^{-3-1}y^5}{5}$

$\frac{-2x^{-4}y^2}{5}$

$\boxed{-\frac{2y^2}{5x^4}}$

2. $(4a^5b^{-2})^{-3}$

$4^{-3}a^{5\cdot -3}b^{-2\cdot -3}$

$4^{-3}a^{-15}b^6$

$\frac{b^6}{4^3a^{15}}$

$\boxed{\frac{b^6}{64a^{15}}}$

3. $(2r^3s^3)(r^{-7}s^5)$

$2r^{3+-7}s^{3+5}$

$2r^{-4}s^8$

$\boxed{\frac{2s^8}{r^4}}$

2. $\frac{xy^{-1}}{x^2y}\cdot\frac{7x^3}{y^{-4}}$

$\frac{7x^{3+1}y^{-1}}{x^2y^{1+-4}}$

$\frac{7x^4y^{-1}}{x^2y^{-3}}$

$7x^{4-2}y^{-1-(-3)}$

$\boxed{7x^2y^2}$

**Review:**

Recall that a <u>monomial</u> is a number, variable, or a product of numbers and variables. Monomials <u>CANNOT</u> have a <u>negative</u> or <u>fraction</u> exponent.

**Examples:** $2x$, $x^4$, $\frac{1}{2}x^3$, $x$ NOT: $2^x$, $x^{-1}$, $\frac{3}{x}$, $x^{1/2}$

A <u>polynomial</u> is a monomial or a sum of monomials.

**Examples:** $15x^{12}-2x^6+x^5-1$, $x^2-1$, $3x^2+\pi x-\frac{1}{3}$

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**Notes:**

A polynomial function is a function of the form:

$$f(x) = a_n x^n + a_{n-1} x^{n-1} + \cdots + a_1 x + a_0$$

(exponents are whole numbers and the coefficients are all real numbers).
For this function, $a_n$ is the leading coefficient, $n$ is the degree, and $a_0$ is the constant term

A polynomial function is in standard form
if its terms are written in descending order of exponents form left to right.

| Common Polynomial Functions | | | |
|---|---|---|---|
| **Degree** | **Type** | **Standard Form** | **Example** |
| 0 | Constant | $f(x) = a_0$ | $f(x) = -14$ |
| 1 | Linear | $f(x) = a_1 x + a_0$ | $f(x) = 5x - 7$ |
| 2 | Quadratic | $f(x) = a_2 x^2 + a_1 x + a_0$ | $f(x) = 2x^2 + x - 9$ |
| 3 | Cubic | $f(x) = a_3 x^3 + a_2 x^2 + a_1 x + a_0$ | $f(x) = x^3 - x^2 + 3x$ |
| 4 | Quartic | $f(x) = a_4 x^4 + a_3 x^3 + a_2 x^2 + a_1 x + a_0$ | $f(x) = x^4 - 2x - 1$ |

**Example #1:** Decide whether the function is a polynomial function. If so, write it in standard form and state its degree, type, and leading coefficient.

1. $h(x) = x^4 - \frac{1}{4}x^2 + 3$

Yes, polynomial.
Already in SF!
Degree: 4
Type: Quartic
Leading coefficient: 1

2. $f(x) = 5x^2 + 3x^{-1} - x$

Not a polynomial function. $3x^{-1}$ has a negative exponent. (not a monomial)

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**Example #2:** Use <u>direct substitution</u> to evaluate the polynomial function for the given value of $x$.

1. $f(x) = 2x^4 - 5x^3 - 4x + 8;\ \ x = 2$

$$f(2) = 2(2)^4 - 5(2)^3 - 4(2) + 8$$
$$= 32 - 40 - 8 + 8$$
$$= \boxed{-8}$$

2. $g(x) = x^4 + 2x^3 - 3x^2 - 7;\ \ x = -2$

$$g(-2) = (-2)^4 + 2(-2)^3 - 3(-2)^2 - 7$$
$$= 16 - 16 - 12 - 7$$
$$= \boxed{-19}$$

**You practice:** Use <u>direct substitution</u> to evaluate the polynomial function for the given value of $x$.

1. $h(x) = x^3 - 5x^2 + 6x + 1;\ \ x = 4$

$$h(4) = (4)^3 - 5(4)^2 + 6(4) + 1$$
$$= 64 - 80 + 24 + 1$$
$$= \boxed{9}$$

2. $g(x) = -3x^3 + x^2 - 12x - 5;\ \ x = 2$

$$g(2) = -3(2)^3 + (2)^2 - 12(2) - 5$$
$$= -24 + 4 - 24 - 5$$
$$= \boxed{-49}$$

**Example #3:** Use <u>synthetic substitution</u> to evaluate the polynomial function for the given value of $x$.

1. $f(x) = 2x^4 - 5x^3 - 4x + 8;\ \ x = 2$

x-value

$$\begin{array}{r|rrrrr} & x^4 & x^3 & x^2 & x & c \\ 2 & 2 & -5 & 0 & -4 & 8 \\ & \downarrow & 4 & -2 & -4 & -16 \\ \hline & 2 & -1 & -2 & -8 & \boxed{-8} \end{array} \leftarrow \text{coefficients}$$

$$\boxed{f(2) = -8}$$

2. $g(x) = x^4 + 2x^3 - 3x^2 - 7;\ \ x = -2$

$$\begin{array}{r|rrrrr} & x^4 & x^3 & x^2 & x & c \\ -2 & 1 & 2 & -3 & 0 & -7 \\ & \downarrow & -2 & 0 & 6 & -12 \\ \hline & 1 & 0 & -3 & 6 & \boxed{-19} \end{array}$$

$$\boxed{g(-2) = -19}$$

**You practice:** Use <u>synthetic substitution</u> to evaluate the polynomial function for the given value of $x$.

1. $h(x) = x^3 - 5x^2 + 6x + 1;\ \ x = 4$

$$\begin{array}{r|rrrr} 4 & 1 & -5 & 6 & 1 \\ & \downarrow & 4 & -4 & 8 \\ \hline & 1 & -1 & 2 & \boxed{9} \end{array}$$

$$\boxed{h(4) = 9}$$

2. $g(x) = -3x^3 + x^2 - 12x - 5;\ \ x = 2$

$$\begin{array}{r|rrrr} 2 & -3 & 1 & -12 & -5 \\ & \downarrow & -6 & -10 & -44 \\ \hline & -3 & -5 & -22 & \boxed{-49} \end{array}$$

$$\boxed{g(2) = -49}$$

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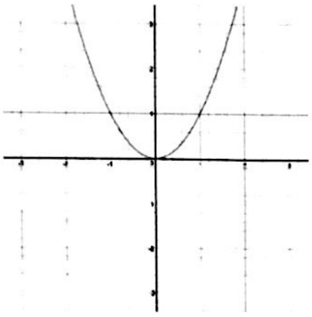
**Exploration #1:** Work with a partner and fill in the following blanks.

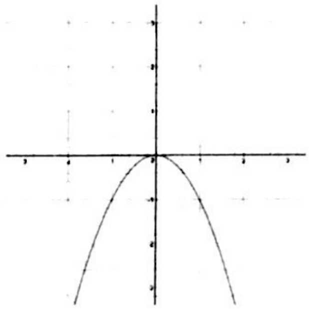
1. $f(x) = x^2$



**Degree:** 2

**Leading coefficient:** 1

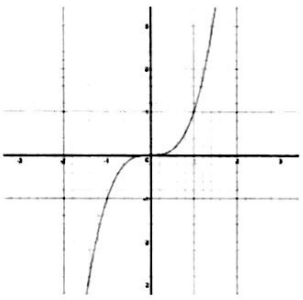
2. $f(x) = -x^2$



**Degree:** 2

**Leading coefficient:** -1

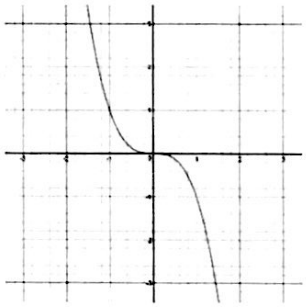
3. $f(x) = x^3$



**Degree:** 3

**Leading coefficient:** 1

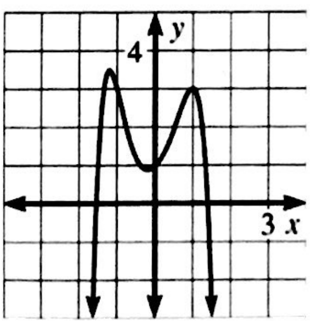
4. $f(x) = -x^3$



**Degree:** 3

**Leading coefficient:** -1

5. **Describe the degree (even or odd) and leading coefficient (positive or negative) of the polynomial function whose graph is shown.**




Degree (circle one): Even Odd

Leading coefficient (circle one): Even Odd

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| End Behavior | | |
|---|---|---|
| Degree ⇒<br>Sign of Leading Coefficient ⇓ | **Degree is EVEN**<br>Ex: $y = -3x^{\boxed{6}} + 3x^3 - 5$ | **Degree is ODD**<br>Ex: $2x^{\boxed{5}} - 3x + 4$ |
| **Leading Coefficient is POSITIVE (+)**<br>Ex: $\boxed{2}x^2 - x + 7$ | $f(x) \to +\infty$ as $x \to -\infty$<br>$f(x) \to +\infty$ as $x \to +\infty$ | $f(x) \to -\infty$ as $x \to -\infty$<br>$f(x) \to +\infty$ as $x \to +\infty$ |
| **Leading Coefficient is NEGATIVE (-)**<br>Ex: $\boxed{-3}x^6 + 3x^3 + 4$ | $f(x) \to -\infty$ as $x \to -\infty$<br>$f(x) \to -\infty$ as $x \to +\infty$ | $f(x) \to -\infty$ as $x \to +\infty$<br>$f(x) \to +\infty$ as $x \to -\infty$ |

**Example #1:** Describe the end behavior of the graph of the polynomial function by completing the statement. Sketch a general picture of the graph to help.

1. $f(x) = 3x^{10} - 16x$

Even degree
+ Coeff.

$f(x) \to$ <u>$+\infty$</u> as $x \to -\infty$

$f(x) \to$ <u>$+\infty$</u> as $x \to +\infty$

2. $f(x) = -2x^3 + 7x - 4$

odd degree
- Coeff.

$f(x) \to$ <u>$+\infty$</u> as $x \to -\infty$

$f(x) \to$ <u>$-\infty$</u> as $x \to +\infty$

**You practice:** Describe the end behavior of the graph of the polynomial function by completing the statement. Sketch a general picture of the graph to help.

1. $f(x) = x^7 + 3x^4 - x^2$

odd degree
+ Coeff.

$f(x) \to$ <u>$-\infty$</u> as $x \to -\infty$

$f(x) \to$ <u>$+\infty$</u> as $x \to +\infty$

2. $f(x) = -5x^8 + 8x^7$

Even degree
- Coeff.

$f(x) \to$ <u>$-\infty$</u> as $x \to -\infty$

$f(x) \to$ <u>$-\infty$</u> as $x \to +\infty$