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NOTES: Section 4.7 – Complete the Square

Goals: #1 - I can solve quadratics by completing the square.

#2 - I can change a quadratic from standard form to vertex form.

Homework: Lesson 4.7 Worksheet

Warm Up:

1. Solve $3x^2 + 8 = -76$

$-8 \quad -8$

$3x^2 = -84$

$x^2 = -28$

$x = \pm\sqrt{-28}$

$x = \pm\sqrt{-1}\cdot\sqrt{4}\cdot\sqrt{7}$

$\boxed{x = \pm 2i\sqrt{7}}$

Write the expression as a complex number in standard form.

2. $(5 - 8i) - (-9 + 3i)$

$5 - (-9) - 8i - 3i$

$\boxed{14 - 11i}$

3. $\frac{5-8i}{-9+3i} \cdot \frac{-9-3i}{-9-3i}$

$\frac{-45 - 15i + 72i + 24i^2}{81 - 9i^2}$

$\frac{-45 + 57i - 24}{81 + 9}$

$\frac{-69}{90} + \frac{57}{90}i$

$\boxed{\frac{-23}{30} + \frac{19}{30}i}$

$i, -1, -i, 1$

$4 \overline{)66}$ R 2

$66 - 64 = 2$

4. $-2 + 55i^{66}$

$-2 + 55(-1)$

$-2 - 55$

$\boxed{-57}$

5. $-10 + 2i^{45}$

$4 \overline{)45}$ R 1

$45 - 44 = 1$

$-10 + 2(i)$

$\boxed{-10 + 2i}$

Exploration #1: Work with a partner.

1. Write some examples of a *perfect square trinomial.*

$x^2 + 6x + 9$

$(x)^2 \quad 2(3x) \quad (3)^2$

* anything: $x^2 + 2(xc) + c^2$

2. What value of c would make the following a perfect square trinomial?

$x^2 + 14x + c$

$(x)^2 \quad 2(x?) \quad (?)^2$

$14x = 2x?$

$? = 7$

$\boxed{c = 49}$

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Example #1: Solve the quadratic equation by finding square roots.

1. $x^2 - 8x + 16 = 25$

$(x)^2 \quad 2(4x) \quad (4)^2$

$(x+4)^2 = 25$

$\sqrt{(x+4)^2} = \pm\sqrt{25}$

$x + 4 = \pm 5$

$x = 4 \pm 5 \rightarrow$ $x = 9$, $x = -1$

2. $x^2 - 10x + 25 = 1$

$(x)^2 \quad 2(5x) \quad (5)^2$

$(x-5)^2 = 1$

$\sqrt{(x-5)^2} = \pm\sqrt{1}$

$x - 5 = \pm 1$

$x = 5 \pm 1$

$x = 6$, $x = 4$

You practice: Solve the quadratic equation by finding square roots.

3. $x^2 + 6x + 9 = 36$

$(x+3)^2 = 36$

$\sqrt{(x+3)^2} = \pm\sqrt{36}$

$x + 3 = \pm 6$

$x = -3 \pm 6 \rightarrow$ $x = -9$, $x = 3$

4. $x^2 - 24x + 144 = 100$

$(x-12)^2 = 100$

$\sqrt{(x-12)^2} = \pm\sqrt{100}$

$x - 12 = \pm 10$

$x = 12 \pm 10$

$x = 22 \quad x = 2$

Example #2: Find the value of c that makes the expression a perfect square trinomial. Then write the expression as the square of a binomial.

1. $x^2 + 16x + c$

$\frac{1}{2}(16) = 8$

$c = (8)^2 = 64$

$(x+8)^2$

2. $x^2 + 14x + c$

$\frac{1}{2}(14) = 7$

$c = (7)^2 = 49$

$(x+7)^2$

You practice: Find the value of c that makes the expression a perfect square trinomial. Then write the expression as the square of a binomial.

3. $x^2 + 22x + c$

$\frac{1}{2}(22) = 11$

$c = (11)^2 = 121$

$(x+11)^2$

4. $x^2 - 9x + c$

$\frac{1}{2}(9) = \frac{9}{2}$

$c = \left(\frac{9}{2}\right)^2 = \frac{81}{4}$

$\left(x - \frac{9}{2}\right)^2$

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Notes:

We can use this idea to make any quadratic expression a perfect square trinomial

This process is called completing the square.

To complete the square for the expression $x^2 + bx$, add $\left(\frac{b}{2}\right)^2$.

$x^2 + bx + \left(\frac{b}{2}\right)^2$ $\quad$ $x^2 - bx + \left(\frac{b}{2}\right)^2$

$\left(x + \frac{b}{2}\right)^2$ $\quad$ $\left(x - \frac{b}{2}\right)^2$

Example #3: Solve the equation by completing the square.

$\left(\frac{b}{2}\right)^2 = \left(\frac{4}{2}\right)^2 = (2)^2 = 4$

1. $x^2 - 12x + 4 = 0$

$\left(\frac{b}{2}\right)^2$ $\left(\frac{-12}{2}\right)^2$ $(-6)^2$ 36

$x^2 - 12x + \square = -4 + \square$

$x^2 - 12x + 36 = -4 + 36$

$(x - 6)^2 = 32$

$x - 6 = \pm\sqrt{32}$

$\boxed{x = 6 \pm 4\sqrt{2}}$

2. $\frac{2x^2 + 8x + 14}{2} = \frac{0}{2}$

$x^2 + 4x + 7 = 0$

$x^2 + 4x + \boxed{4} = -7 + \boxed{4}$

$(x + 2)^2 = -3$

$x + 2 = \pm\sqrt{-3}$

$\boxed{x = -2 \pm i\sqrt{3}}$

You practice: Solve the equation by completing the square.

$\left(\frac{b}{2}\right)^2$ $\left(\frac{-10}{2}\right)^2$ $(-5)^2$ 25

3. $x^2 - 10x + 8 = 0$

$x^2 - 10x + \boxed{25} = -8 + \boxed{25}$

$(x - 5)^2 = 17$

$x - 5 = \pm\sqrt{17}$

$\boxed{x = 5 \pm \sqrt{17}}$

$\left(\frac{4}{2}\right)^2 = (2)^2 = 4$

4. $\frac{3x^2 + 12x - 18}{3} = \frac{0}{3}$

$x^2 + 4x - 6 = 0$

$x^2 + 4x + \boxed{4} = 6 + \boxed{4}$

$(x + 2)^2 = 10$

$x + 2 = \pm\sqrt{10}$

$\boxed{x = -2 \pm \sqrt{10}}$

Notes:

Recall the vertex form of a quadratic function is $y = a(x - h)^2 + k$.

We use completing the square to write any quadratic function in vertex form.

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Example #4: Write the quadratic function in vertex form. Then identify the vertex.

1. $y = x^2 - 10x + 22$ $\left(\frac{-10}{2}\right)^2 = (-5)^2 = 25$

$25 + y = x^2 - 10x + 25 + 22$

$25 + y = (x-5)^2 + 22$

$y = (x-5)^2 - 3$

vertex: $(5, -3)$

2. $y = x^2 - 8x + 17$ $\left(\frac{-8}{2}\right)^2 = (-4)^2 = 16$

$16 + y = x^2 - 8x + 16 + 17$

$16 + y = (x-4)^2 + 17$

$y = (x-4)^2 + 1$

vertex: $(4, 1)$

You practice: Write the quadratic function in vertex form. Then identify the vertex.

3. $y = x^2 + 6x + 3$ $\left(\frac{6}{2}\right)^2 = (3)^2 = 9$

$9 + y = x^2 + 6x + 9 + 3$

$9 + y = (x+3)^2 + 3$

$y = (x+3)^2 - 6$

vertex: $(-3, -6)$

4. $y = x^2 - 4x - 4$ $\left(\frac{-4}{2}\right)^2 = (-2)^2 = 4$

$4 + y = x^2 - 4x + 4 - 4$

$y = (x-2)^2 - 4$

$y = (x-2)^2 - 8$

vertex: $(2, -8)$

Example #5: The height y (in feet) of a baseball t seconds after it is hit is given by this function: $y = -16t^2 + 96t + 3$. Find the maximum height of the baseball.

$y = -16(t^2 - 6t) + 3$ vertex! $\left(\frac{-6}{2}\right)^2 = (-3)^2 = 9$

$-144 + y = -16(t^2 - 6t + 9) + 3$

$-144 + y = -16(t-3)^2 + 3$

$y = -16(t-3)^2 + 147$

vertex: $(3, 147)$

147 ft → max height