Name: KEY Hour: ____________ Date: ____________

NOTES: Section 10.2 – Use Combinations and the Binomial Theorem

Goals: #1 - I can use combinations to count.

#2 - I can use "AND" and "OR" to evaluate a combination.

#3 - I can use apply the binomial theorem.

Homework: Lesson 10.2 Worksheet

Warm Up:

1. Find the number of distinguishable permutations of the letters in the word BASKETBALL.

$\frac{10!}{2!\,2!\,2!} = \boxed{453,600}$

2. How many different 3-digit IDs can be made if the first digit must be a 7 and no digits may be repeated?

$7 \quad 9 \cdot 8 = \boxed{72}$

3. How many different ways can a chairperson and an assistant be selected for a research project if there are 7 qualified scientists for the position?

${}_7P_2$ OR $7 \cdot 6 = \boxed{42}$

Example #1:

There are 6 people that need to be seated. In how many ways can these 6 peoples be seated in 3 chairs?

$6 \cdot 5 \cdot 4 = \boxed{120}$

What if... we only cared about choosing 3 people to sit down, but we didn't care about what order they are sitting. In how many ways can I choose 3 out of the 6 people where I don't care which chair they sit on? A, B, C, D, E, F

ABC BAC CAB DEF EDF FED
ACB BCA CBA DFE EFD FDE } same combination

6 permutations
1 combination

$\frac{120}{6} = \boxed{20 \text{ combinations}}$

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Notes:

In Section 10.1, we learned that order is important for some counting problems.

A permutation is a selection of r objects from a group of n objects where ORDER IS IMPORTANT!: $_nP_r = \frac{n!}{(n-r)!}$

Example: ABC is a different permutation than ACB

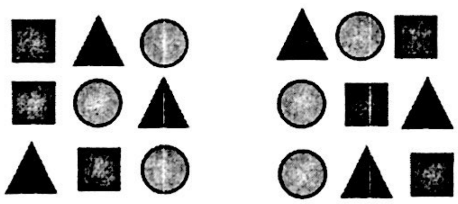
For other counting problems, order is NOT important.

A combination is a selection of r objects from a group of n objects where ORDER IS NOT IMPORTANT!: $_nC_r = \frac{n!}{(n-r)!r!}$

Example: ABC is the same combination as ACB

Permutations vs. Combinations

Permutations



Combinations

Example #2:

You are picking 7 books from a stack of 32. How many different 7 book groups are possible?

(order doesn't matter - still the same books)

$_{32}C_7 = \boxed{3{,}365{,}856 \text{ combinations}}$

You practice:

A relay race has a team of 4 runners who run different parts of the race. There are 20 students on your track squad. In how many ways can the coach select students to compete on the relay team?

$_{20}C_4 = \boxed{4845 \text{ combinations}}$

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Example #3:

A standard deck of 52 playing cards has 4 suits with 13 different cards in each suit.

Standard 52-Card Deck

Spades	Hearts	Diamonds	Clubs
K ♠	K ♥	K ♦	K ♣
Q ♠	Q ♥	Q ♦	Q ♣
J ♠	J ♥	J ♦	J ♣
10 ♠	10 ♥	10 ♦	10 ♣
9 ♠	9 ♥	9 ♦	9 ♣
8 ♠	8 ♥	8 ♦	8 ♣
7 ♠	7 ♥	7 ♦	7 ♣
6 ♠	6 ♥	6 ♦	6 ♣
5 ♠	5 ♥	5 ♦	5 ♣
4 ♠	4 ♥	4 ♦	4 ♣
3 ♠	3 ♥	3 ♦	3 ♣
2 ♠	2 ♥	2 ♦	2 ♣
A ♠	A ♥	A ♦	A ♣

a. If the order in which the cards are dealt is not important, haw many different 5-card hands are possible?

$_{52}C_5 = \boxed{2{,}598{,}960 \text{ hands}}$

b. In how many 5-card hands are all 5 cards face cards (kings, queens, or jacks)?

$_{12}C_5 = \boxed{792 \text{ hands}}$

Notes:

When finding the number of ways both an event *A* AND event *B* can occur, you need to multiply the number of ways.

Example: Find the number of possible 5-card hands that contain 4 kings <u>and</u> 1 card that is not a king.

$_4C_4 \cdot {}_{48}C_1$

$1 \cdot 48 = \boxed{48 \text{ hands}}$

When finding the number of ways that event *A* OR event *B* can occur, you need to add the number of ways.

Example: Find the number of possible 5-card hands that contain 5 hearts <u>or</u> 5 diamonds.

$_{13}C_5 + {}_{13}C_5$

$1287 + 1287 = \boxed{2574 \text{ hands}}$

Couting problems often involve phrases that contain at least or at most. In some cases, subtracting may be easier.

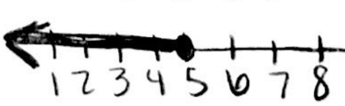
0Q 5OC
1Q 4OC
2Q 3OC
3Q 2OC
4Q 1OC
5Q 0OC

Example: Find the number of possible 5-card hands that contain at least 1 queen.

$_{52}C_5 - ({}_4C_0 \cdot {}_{48}C_5) = 886{,}656$ hands

Total − [0 Queens ∶ 5 OC]

- At least 5:



- At most 5:

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You practice:

NOT 0G 1G 2G

During the school year, the girls' basketball team is scheduled to play 12 home games. You want to attend at least 3 of the games. How many different combinations of the games can you attend?

3G 4G 5G 6G 7G 8G 9G 10G 11G 12G

Total − [0G OR 1G OR 2G]

attend OR Not attend → $2^{12} - ({}_{12}C_0 + {}_{12}C_1 + {}_{12}C_2)$

$= 4096 - 79 = \boxed{4017 \text{ combinations}}$

Notes:

If you arrange the values of ___combinations___ in a triangular pattern, you get what is called ___Pascals Triangles___.

	Pascal's triangle as combinations	**Pascal's triangle as numbers**
$n = 0$ (0th row)	${}_0C_0$	1
$n = 1$ (1st row)	${}_1C_0 \quad {}_1C_1$	1 1
$n = 2$ (2nd row)	${}_2C_0 \quad {}_2C_1 \quad {}_2C_2$	1 2 1
$n = 3$ (3rd row)	${}_3C_0 \quad {}_3C_1 \quad {}_3C_2 \quad {}_3C_3$	1 3 3 1
$n = 4$ (4th row)	${}_4C_0 \quad {}_4C_1 \quad {}_4C_2 \quad {}_4C_3 \quad {}_4C_4$	1 4 6 4 1
$n = 5$ (5th row)	${}_5C_0 \quad {}_5C_1 \quad {}_5C_2 \quad {}_5C_3 \quad {}_5C_4 \quad {}_5C_5$	1 5 10 10 5 1

The numbers in Pascal's triangle can be used to find ___coefficients___ in binomial expansion.

Example #4: Use the binomial theorem to write the binomial expansion.

1. $(x^2 + y)^3$ $n = 3$

$1(x^2)^3(y)^0 + 3(x^2)^2(y)^1 + 3(x^2)^1(y)^2 + 1(x^2)^0(y)^3$

$\boxed{x^6 + 3x^4y + 3x^2y^2 + y^3}$

2. $(2p - q)^2$ $n = 2$

$1(2p)^2(-q)^0 + 2(2p)^1(-q)^1 + 1(2p)^0(-q)^2$

$\boxed{4p^2 - 4pq + q^2}$