Name: KEY Hour: ____________ Date: ____________

NOTES: Section 5.5 – Apply the Remainder and Factor Theorems

Goals: #1 - I can perform polynomial long division.

#2 - I can divide polynomials synthetically.

#3 - I can factor a 3rd degree polynomial when one factor is given.

#4 - I can find the zeros of a 3rd degree polynomial when one zero is given.

Homework: Lesson 5.5 Worksheet

Exploration #1: Work with a partner and answer the following questions.

1. Use long division to find the following quotients.

a. Divide 258 by 6.

$$6\overline{)258} = 43$$

$$\begin{array}{r} 258 \\ -24 \\ \hline 18 \\ -18 \\ \hline 0 \end{array}$$

b. Divide 1122 by 17.

$$17\overline{)1122} = 66$$

$$\begin{array}{r} 1122 \\ -102 \\ \hline 102 \\ -102 \\ \hline 0 \end{array}$$

c. Divide 289 by 8.

$$8\overline{)289} = 36$$

$$\begin{array}{r} 289 \\ -24 \\ \hline 49 \\ -48 \\ \hline R1 \end{array}$$

$36\frac{1}{8}$

d. Divide 1704 by 18.

$$18\overline{)1704} = 94$$

$$\begin{array}{r} 1704 \\ -162 \\ \hline 84 \\ -72 \\ \hline 12 \end{array}$$

$94\frac{12}{18}$

$94\frac{2}{3}$

Notes:

One way to divide polynomials is called polynomial long division

Example #1: Divide using polynomial long division.

1. $(x^3 + 5x^2 - 7x + 2) \div (x - 2)$

$$x - 2\overline{)x^3 + 5x^2 - 7x + 2} = x^2 + 7x + 7$$

$$\begin{array}{r} x^3 + 5x^2 - 7x + 2 \\ -(\cancel{x^3} - 2x^2) \\ \hline \cancel{7x^2} - 7x \\ -(\cancel{7x^2} - 14x) \\ \hline 7x + 2 \\ -(7x - 14) \\ \hline 16R \end{array}$$

$$\boxed{x^2 + 7x + 7 + \frac{16}{x-2}}$$

2. $(3x^4 - 5x^3 + 4x - 6) \div (x^2 - 3x + 5)$

$$x^2 - 3x + 5\overline{)3x^4 - 5x^3 + 0x^2 + 4x - 6} = 3x^2 + 4x - 3$$

$$\begin{array}{r} 3x^4 - 5x^3 + 0x^2 + 4x - 6 \\ -(\cancel{3x^4} - 9x^3 + 15x^2) \\ \hline \cancel{4x^3} - 15x^2 + 4x \\ -(\cancel{4x^3} - 12x^2 + 20x) \\ \hline -\cancel{3x^2} - 16x - 6 \\ -(-\cancel{3x^2} + 9x - 15) \\ \hline -25x + 9\ R \end{array}$$

$$\boxed{3x^2 + 4x - 3 + \frac{-25x + 9}{x^2 - 3x + 5}}$$

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You practice: Divide using polynomial long division.

1. $(2x^4 + x^3 + x - 1) \div (x^2 + 2x - 1)$

$$
\begin{array}{r}
2x^2 - 3x + 8 \\
x^2+2x-1 \overline{)\, 2x^4 + x^3 + 0x^2 + x - 1} \\
-\ (\cancel{2x^4} + 4x^3 - 2x^2) \downarrow \\
\hline
-\cancel{3x^3} + 2x^2 + x \\
-\ (-3x^3 - 6x^2 + 3x) \\
\hline
8x^2 - 2x - 1 \\
-\ (8x^2 + 16x - 8) \\
\hline
-18x + 7 \ R
\end{array}
$$

$\boxed{2x^2 - 3x + 8 + \frac{-18x+7}{x^2+2x-1}}$

2. $(x^3 - x^2 + 4x - 10) \div (x + 2)$

$$
\begin{array}{r}
x^2 - 3x + 10 \\
x+2 \overline{)\, \cancel{x^3} - x^2 + 4x - 10} \\
-\ (\cancel{x^3} + 2x^2) \downarrow \\
\hline
-\cancel{3x^2} + 4x \\
-\ (-3x^2 - 6x) \\
\hline
\cancel{10x} - 10 \\
-\ (10x + 20) \\
\hline
-30 \ R
\end{array}
$$

$\boxed{x^2 - 3x + 10 + \frac{-30}{x+2}}$

Notes:

Another way to divide polynomials is called synthetic division

THIS ONLY WORKS WHEN DIVIDING BY A LINEAR POLYNOMIAL!

$\div (x-2)$ $\quad$ * $x - k$

$\div (x+1)$

Example #2: Divide using synthetic division.

1. $(x^3 + 5x^2 - 7x + 2) \div (x - \ \ )$

$$
\begin{array}{r|rrrr}
2 & 1 & 5 & -7 & 2 \\
 & \downarrow & 2 & 14 & 14 \\
\hline
 & 1 & 7 & 7 & \boxed{16}\ R
\end{array}
$$

$\boxed{x^2 + 7x + 7 + \frac{16}{x-2}}$

2. $(2x^3 + x^2 - 8x + 5) \div (x + 3)$

$$
\begin{array}{r|rrrr}
-3 & 2 & 1 & -8 & 5 \\
 & \downarrow & -6 & 15 & -21 \\
\hline
 & 2 & -5 & 7 & \boxed{-16}\ R
\end{array}
$$

$\boxed{2x^2 - 5x + 7 + \frac{-16}{x+3}}$

You practice: Divide using synthetic division.

1. $(x^3 - x^2 + 4x - 10) \div (x + 2)$

$$
\begin{array}{r|rrrr}
-2 & 1 & -1 & 4 & -10 \\
 & \downarrow & -2 & 6 & -20 \\
\hline
 & 1 & -3 & 10 & \boxed{-30}\ R
\end{array}
$$

$\boxed{x^2 - 3x + 10 + \frac{-30}{x+2}}$

2. $(4x^3 + x^2 - 3x + 7) \div (x - 1)$

$$
\begin{array}{r|rrrr}
1 & 4 & 1 & -3 & 7 \\
 & \downarrow & 4 & 5 & 2 \\
\hline
 & 4 & 5 & 2 & \boxed{9}\ R
\end{array}
$$

$\boxed{4x^2 + 5x + 2 + \frac{9}{x-1}}$

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Exploration #2: Work with a partner and answer the following questions.

1. Factor the polynomial $x^3 + 5x^2 - 9x - 45$ completely.

$x^2(x+5) - 9(x+5)$

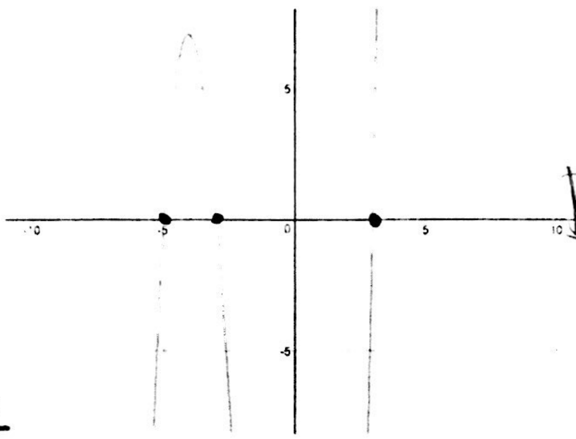
$(x+5)(x^2-9)$

$(x+5)(x-3)(x+3)$

2. Below is a snapshot of the graph $f(x) = x^3 + 5x^2 - 9x - 45$. Find the zeros.

$$\begin{array}{r|rrrr} 3 & 1 & 5 & -9 & -45 \\ & \downarrow & 3 & 24 & 45 \\ \hline & 1 & 8 & 15 & \boxed{0} \text{ R} \end{array}$$

$$\begin{array}{r|rrrr} -5 & 1 & 5 & -9 & -45 \\ & \downarrow & -5 & 0 & 45 \\ \hline & 1 & 0 & -9 & \boxed{0} \text{ R} \end{array}$$



$\boxed{x = -5, -3, 3}$

3. Find the real-number solutions of the equation $x^3 + 5x^2 - 9x - 45 = 0$

$(x+5)(x-3)(x+3) = 0$

$\boxed{x = -5}$ $\boxed{x = 3}$ $\boxed{x = -3}$

Notes:

- Factor Theorem:

A polynomial $f(x)$ has a factor $x-k$ if and only if $f(k) = 0$.

Example: factor of $(x-2)$? only if $f(2) = 0$

The factor therom can be used to solve a variety of problems.

- Given one factor of a polynomial, find the other factors.
- Give one zero of a polynomial function, find the other zeros.
- Given one solution of a polynomial equation, find the other solutions.

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Example #3: Factor $f(x) = 3x^3 - 4x^2 - 28x - 16$ completely given that $x + 2$ is a factor.

* use synthetic division

$f(-2) = 0$

$$\begin{array}{r|rrrr} -2 & 3 & -4 & -28 & -16 \\ & \downarrow & -6 & 20 & 16 \\ \hline & 3 & -10 & -8 & 0 \end{array}$$

$3x^2 - 10x - 8$

$f(x) = 3x^3 - 4x^2 - 28x - 16$ -24: -12, 2

$f(x) = (x+2)(3x^2 - 10x - 8)$

$f(x) = (x+2)(3x^2 - 12x + 2x - 8)$

$f(x) = (x+2)(3x(x-4) + 2(x-4))$

$\boxed{f(x) = (x+2)(x-4)(3x+2)}$

You practice: Factor $f(x) = x^3 - 6x^2 + 5x + 12$ completely given that $x - 4$ is a factor.

$f(4) = 0$

$$\begin{array}{r|rrrr} 4 & 1 & -6 & 5 & 12 \\ & \downarrow & 4 & -8 & -12 \\ \hline & 1 & -2 & -3 & 0 \end{array}$$

$x^2 - 2x - 3$

$f(x) = (x-4)(x^2 - 2x - 3)$

$\boxed{f(x) = (x-4)(x-3)(x+1)}$

Example #4: Find the other zeros of the function $f(x) = x^3 - 2x^2 - 23x + 60$ given that 3 is zero. $(x-3)$

* use synthetic division

$$\begin{array}{r|rrrr} 3 & 1 & -2 & -23 & 60 \\ & \downarrow & 3 & 3 & -60 \\ \hline & 1 & 1 & -20 & \boxed{0}\ R \end{array}$$

$x^2 + x - 20$

$f(x) = (x-3)(x^2 + x - 20)$

$0 = (x-3)(x+5)(x-4)$

$x - 3 = 0$ $x + 5 = 0$ $x - 4 = 0$

$\boxed{x = 3}$ $\boxed{x = -5}$ $\boxed{x = 4}$

given

You practice: Find the other zeros of the function $f(x) = x^3 + 8x^2 + 5x - 14$ given that -2 is zero.

$(x+2)$

$$\begin{array}{r|rrrr} -2 & 1 & 8 & 5 & -14 \\ & \downarrow & -2 & -12 & 14 \\ \hline & 1 & 6 & -7 & \boxed{0}\ R \end{array}$$

$x^2 + 6x - 7$

$f(x) = (x+2)(x^2 + 6x - 7)$

$f(x) = (x+2)(x+7)(x-1)$

$x + 2 = 0$ $x + 7 = 0$ $x - 1 = 0$

$\boxed{x = -2}$ $\boxed{x = -7}$ $\boxed{x = 1}$