Name: KEY Hour: ____________ Date: ____________

NOTES: Section 4.6 – Perform Operations with Complex Numbers

Goals: #1 - I can solve equations that have both real and imaginary solutions (by finding square roots).

#2 - I can add, subtract, multiply, and divide complex numbers.

#3 - I can use the properties of exponents to write a complex number in standard form.

Homework: Lesson 4.6 Worksheet

Exploration #1:

1. Solve $2x^2 + 11 = -37$

$-11 \quad -11$

$\frac{2x^2}{2} = \frac{-48}{2}$

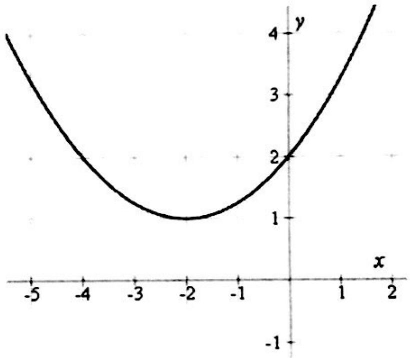
$x^2 = -24$

$\sqrt{x^2} = \pm\sqrt{-24}$

??

what

2. Look at the graph below.



a. What are the x-intercepts? NONE!

Notes:

Not all quadratic equations have real number solutions.

Mathematicians created a system of numbers using the imaginery unit i, defined as $i = \sqrt{-1}$. So $i^2 = (\sqrt{-1})^2 = -1$ ★

The Square Root of a Negative Number:

$\sqrt{-1}\cdot\sqrt{3}$

- If r is a positive real number, then $\sqrt{-r} = i\sqrt{r}$. **Example:** $\sqrt{-3} = i\sqrt{3}$
- By the above property, it follows that $(i\sqrt{r})^2 = -r$. **Example:** $(i\sqrt{3})^2 = i^2 \cdot 3$ $= -3$

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Example #1: Solve the following quadratic equations.

1. $2x^2 + 11 = -37$

$-11 \quad -11$

$\frac{2x^2}{2} = \frac{-48}{2}$

$x^2 = -24$

$\sqrt{x^2} = \pm\sqrt{-24}$

$x = \pm\sqrt{-1}\cdot\sqrt{4}\cdot\sqrt{6}$

$i \quad 2 \quad \sqrt{6}$

$\boxed{x = \pm 2i\sqrt{6}}$

2. $5x^2 + 33 = 3$

$-33 \quad -33$

$5x^2 = -30$

$x^2 = -6$

$\sqrt{x^2} = \pm\sqrt{-6}$

$x = \pm\sqrt{-1}\cdot\sqrt{6}$

$\boxed{x = \pm i\sqrt{6}}$

You practice: Solve the following quadratic equations.

3. $3x^2 - 7 = -31$

$+7 \quad +7$

$\frac{3x^2}{3} = \frac{-24}{3}$

$x^2 = -8$

$\sqrt{x^2} = \pm\sqrt{-8}$

$x = \pm\sqrt{-1}\cdot\sqrt{4}\cdot\sqrt{2}$

$i \quad 2 \quad \sqrt{2}$

$\boxed{x = \pm 2i\sqrt{2}}$

4. $x^2 + 11 = 3$

$-11 \quad -11$

$x^2 = -8$

$\sqrt{x^2} = \pm\sqrt{-8}$

$x = \pm\sqrt{-1}\cdot\sqrt{4}\cdot\sqrt{2}$

$\boxed{x = \pm 2i\sqrt{2}}$

Notes:

A complex number is a number a + bi where a and b are real numbers

If $b \neq 0$, then a + bi is an imaginary number

real part ↓ imaginary part ↓

Standard Form: a + bi

Examples: 5 − 3i → imaginary #

−1 + 0i = −1 → real #

Complex Numbers ($a + bi$)

Real Numbers ($a + 0i$)	Imaginary Numbers ($a + bi, b \neq 0$)
	$2 + 3i$ $\quad 5 - 5i$
$-1 \quad \frac{5}{2}$	**Pure Imaginary Numbers** ($0 + bi, b \neq 0$)
$\pi \quad \sqrt{2}$	$-4i \quad 6i$

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Notes:

To add or subtract two complex numbers, add or subtract their real parts and their imaginary parts separately.

Example #2: Write the expression as a complex number in standard form.

1. $(8 - i) + (5 + 4i)$

$8+5 \quad -i+4i$

$\boxed{13+3i}$

2. $(7 - 6i) - (3 - 6i)$

$7-3 \quad -6i+6i$

$4+0i$

$\boxed{4}$

You practice: Write the expression as a complex number in standard form.

3. $(9 - i) + (-6 + 7i)$

$9-6 \quad -i+7i$

$\boxed{3+6i}$

4. $10 - (6 + 7i) + 4i$

$10-6-7i+4i$

$\boxed{4-3i}$

Notes:

To multiply two complex numbers, use the distributive method or FOIL

Just as you do when multiplying real numbers or algebraic expressions.

Example #3: Write the expression as a complex number in standard form.

1. $4i(-6 + i)$

$-24i+4i^2$

$-24i+4(-1)$

$\boxed{-4-24i}$

2. $(9 - 2i)(-4 + 7i)$

$9(-4)+9(7i)-2i(-4)-2i(7i)$

$-36+63i+8i-14i^2$

$-36+71i-14(-1)$

$-36+71i+14$

$\boxed{-22+71i}$

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You practice: Write the expression as a complex number in standard form.

3. $i(9-i)$

$9i - i^2$

$9i - (-1)$

$9i + 1$

$\boxed{1 + 9i}$

4. $(3+i)(5-i)$

$3(5) - 3i + 5i - i^2$

$15 + 2i - (-1)$

$15 + 2i + 1$

$\boxed{16 + 2i}$

Notes:

Two complex numbers of the form $a + bi$ and $a - bi$ are called complex conjugates.

The product of $(a+bi)(a-bi)$ is always a real number.

Example: $(4+3i)(4-3i)$

F O I L

$16 - 12i + 12i - 9i^2$

$16 - 9(-1) = 16 + 9 = \boxed{25}$

We use this to divide complex numbers.

Example #4: Write the expression as a complex number in standard form.

1. $\frac{7+5i}{1-4i} \cdot \frac{(1+4i)}{1+4i}$

$\frac{7 + 28i + 5i + 20i^2}{1 - 16i^2}$

$\frac{7 + 33i + 20(-1)}{1 - 16(-1)}$

$\frac{7 + 33i - 20}{1 - (-16)}$

$\frac{-13 + 33i}{17}$

$\boxed{\frac{-13}{17} + \frac{33}{17}i}$

2. $\frac{5}{1+i} \cdot \frac{1-i}{1-i}$

$\frac{5 - 5i}{1 - i^2}$

$\frac{5 - 5i}{1 - (-1)}$

$\frac{5 - 5i}{2}$

$\boxed{\frac{5}{2} - \frac{5}{2}i}$

You practice: Write the expression as a complex number in standard form.

3. $\frac{5+2i}{3-2i} \cdot \frac{3+2i}{3+2i}$

$\frac{15 + 10i + 6i + 4i^2}{9 - 4i^2}$

$\frac{15 + 16i + 4(-1)}{9 - 4(-1)}$

$\frac{15 + 16i - 4}{9 - (-4)}$

$\frac{11 + 16i}{13}$

$\boxed{\frac{11}{13} + \frac{16}{13}i}$

4. $\frac{7i}{8+i} \cdot \frac{8-i}{8-i}$

$\frac{56i - 7i^2}{64 - i^2}$

$\frac{56i - 7(-1)}{64 - (-1)}$

$\frac{56i + 7}{65}$

$\boxed{\frac{7}{65} + \frac{56}{65}}$

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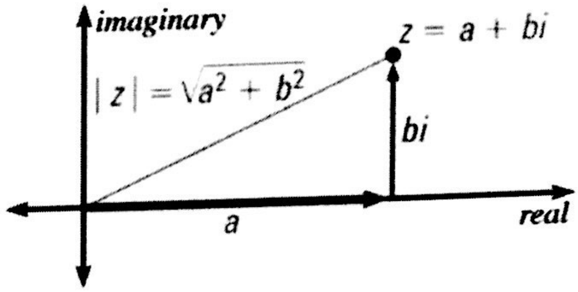
Notes:

The absolute value of a complex number $z = a + bi$, denoted $|z|$ is a nonnegative real number defined as $\sqrt{a^2+b^2}$.

This is the distance between z and the origin in the complex plane.



Example #5: Find the absolute value of the complex number.

1. $-4 + 3i$

a b

$|-4+3i| = \sqrt{(-4)^2+(3)^2}$

$= \sqrt{16+9}$

$= \sqrt{25}$

$= \boxed{5}$

2. $-3i$

$|-3i| = \sqrt{(0)^2+(-3)^2}$

$= \sqrt{9}$

$= \boxed{3}$

You practice: Write the expression as a complex number in standard form.

3. $-3 - 4i$

a b

$|-3-4i| = \sqrt{(-3)^2+(-4)^2}$

$= \sqrt{9+16}$

$= \sqrt{25}$

$= \boxed{5}$

4. $2 + 5i$

a b

$|2+5i| = \sqrt{(2)^2+(5)^2}$

$= \sqrt{4+25}$

$= \boxed{\sqrt{29}}$

Notes:

We can raise the the imaginary unit, i to different powers to notice a pattern.

- $i = \sqrt{-1} = i$
- $i^2 = -1$
- $i^3 = i^2 \cdot i = -i$
- $i^4 = i^3 \cdot i = 1$

$-i \cdot i$

$-(-1)$

1

- $i^5 = i^4 \cdot i = i$
- $i^6 = i^5 \cdot i = -1$
- $i^7 = i^6 \cdot i = -i$
- $i^8 = i^7 \cdot i = 1$

- $i^9 = i$
- $i^{10} = -1$
- $i^{11} = -i$
- $i^{12} = 1$

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Example #6: Using the properties of exponents, write the complex number in standard form.

1. $-2 + i^2$

$-2 + (-1)$

$\boxed{-3}$

2. $1 - 5i^7$

$1 - 5(-i)$

$\boxed{1 + 5i}$

You practice: Write the expression as a complex number in standard form.

3. $2 - i^8$

$2 - (1)$

$\boxed{1}$

4. $5 + i^3$

$5 + (-i)$

$\boxed{5 - i}$

CHALLENGE: What would i^{39} be? What about i^{101}

9 R3

$4 \overline{)39}$

36

3

$i^{39} = -i$

25 R1

$4 \overline{)101}$

8

21

$i^{101} = i$