Name: KEY Hour: ____________ Date: ____________

NOTES: Section 10.1 – Apply the Counting Principle and Permutations

Goals: #1 - I can apply the fundamental counting principle.

#2 - I can use permutations to count.

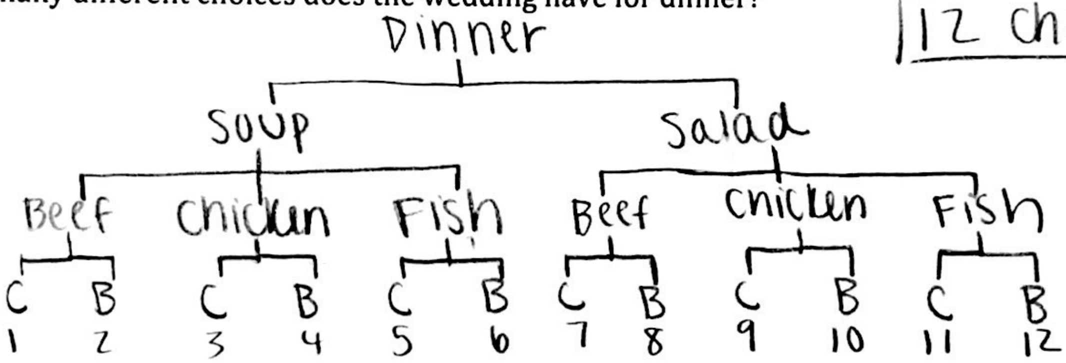
#3 - I can use permutations with repetition to count.

Homework: Lesson 10.1 Worksheet

Example #1:

For a wedding dinner, you have a choice between soup or salad. You also have your choice of entree between beef, chicken, or fish. Lastly, you can choose between cake of a brownie. How many different choices does the wedding have for dinner?

12 choices



Notes:

In many real-life problems, we can count the number of ways to perform a task. One way to do this is to use a tree diagram.

Another way to count the number of ways to perform a task is by using the Fundamental Counting Principle

- 2 Events: If one event can occur in m ways and another event can occur in n ways, then the number of ways that both events occur is $m \cdot n$.
- 3 or more Events: If three events can occur in m, n and p ways, then the number of ways that all three events occur is $m \cdot n \cdot p$.

Ex #1: Using FCP, $2 \cdot 3 \cdot 2 =$ 12 choices

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Example #2:

You are buying a pizza. You have a choice of 3 crusts, 4 cheeses, 5 meat toppings, and 8 vegetable toppings. Have many different pizzas with one crust, one cheese, one meat, and one vegetable can you choose?

$\underset{CR}{3} \cdot \underset{CH}{4} \cdot \underset{M}{5} \cdot \underset{V}{8}$ = 480 different pizzas

Example #3:

The new configuration for a Wisconsin license plate is 3 letters followed by 4 digits.

a. How many different license plates are possible if letters and digits can be repeated?

$26 \cdot 26 \cdot 26 \cdot 10 \cdot 10 \cdot 10 \cdot 10$

= 175,760,000 possibilities

b. How many different license plates are possible if letter and digits cannot be repeated?

$26 \cdot 25 \cdot 24 \cdot 10 \cdot 9 \cdot 8 \cdot 7$

= 78,624,000 possibilities

You practice:

A town has telephone numbers that all begin with 329 followed by four digits.

a. How many different phone numbers are possible if the numbers can be repeated?

329 – $10 \cdot 10 \cdot 10 \cdot 10$

= 10,000 possibilities

b. How many different phone numbers are possible if the numbers cannot be repeated?

329 – $7 \cdot 6 \cdot 5 \cdot 4$

= 840 possibilities

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Notes:

An ordering of n number of objects is a permutation.

Example: How many different ways can the letters A, B, and C be arranged?

ABC ACB BAC BCA CAB CBA

6 permutations   OR $\underline{3} \cdot \underline{2} \cdot \underline{1} = 6$

The expression $3 \cdot 2 \cdot 1$ can also be written as 3!. In mathematics, the symbol ! is called a factorial.

Example: 7! $7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 5040$

Calculator: $7! = 5040$

The number of permutations of n objects is $n!$.

Example #4:

Eight teams are competing in a baseball playoff.

a. In how many different ways can the baseball teams finish the competition?

$\underline{8} \cdot \underline{7} \cdot \underline{6} \cdot \underline{5} \cdot \underline{4} \cdot \underline{3} \cdot \underline{2} \cdot 1$ OR 8!

= 40,320 ways

b. In how many different ways can the baseball teams finish first, second, and third?

$\underline{8} \cdot \underline{7} \cdot \underline{6}$

= 336 ways

Notes:

The answer to part (b.) is called the number of permutations of 8 objects taken 3 at a time. This is denoted by: $_8P_3$

Calculator: $_8P_3 = 336$

In gneral, the number of permutations of r objects taken from a group of n distinct objects is denoted by $_nP_r$ and given by this formula: $_nP_r = \frac{n!}{(n-r)!}$

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Calculator practice:

1. $_8P_3$ **336**
2. $_{12}P_4$ **11880**
3. $_{15}P_0$ **1**

Example #5:

You have 6 homework assignments to complete over the weekend. However, you only have time to complete 4 of them on Saturday. In how many orders can you complete 4 of the assignments?

$_6P_4$ OR $6 \cdot 5 \cdot 4 \cdot 3$

$= $ **360 orders**

Notes:

The number of distinguishable permutations of n objects where one object is repeated s_1 times, another is repeated s_2 times is given by the formula: $\frac{n!}{s_1! \cdot s_2! \cdot \ldots \cdot s_k!}$

Example #6: Find the number of distinguishable permutations of the letters in

a. EVEN $\frac{4!}{2!} = $ **12**

$n = 4$
$s_1 = E = 2$

b. CALIFORNIA $\frac{10!}{2! \cdot 2!} = $ **907,200**

$n = 10$
$s_1 = A = 2$
$s_2 = I = 2$

You practice:

1. Have many different ways can 4 raffle tickets be selected from 50 tickets if each ticket wins a different prize? $_{50}P_4$ OR $50 \cdot 49 \cdot 48 \cdot 47$

   $= $ **5,527,200**

2. Find the number of distinguishable permutations of the letters in CINCINNATI.

   $\frac{10!}{2!\,3!\,3!}$

   $= $ **50,400**

   $n = 10$
   $s_1 = C = 2$
   $s_2 = I = 3$
   $s_3 = N = 3$